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PARAMETRIC ANALYSIS OF PRESSURE VARIATION OF FLUID IN HYDRODYNAMIC JOURNAL BEARING

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ABSTRACT

In this work, a parametric study of the pressure variation of fluid (lubricant) in a hydrodynamic journal bearing was carried out using the finite element (FEM) method. The classical Reynolds equation was used to represent the pressure behaviour. The domain of the bearing was discretized and pressure analysis was carried out on these finite elements. The overall results from these finite elements were finally assembled to represent the pressure distribution in the entire domain of the bearing. The result obtained showed that the pressure increased from the ambient pressure which was taken to be zero to its maximum value at an angular displacement of 145.37^o. At this point, the pressure had a maximum value of 1.955 MPa and then drop until it returned to the ambient value at 180^o. Between angular displacements of 180 and 360^o, there were also pressure variations which were equal in magnitude with those between 0 and 360^o but with different directions. The result obtained shows a strong positive correlation with existing results in literature.

1. INTRODUCTION

Journal bearings have been widely used for a long time but their lubrication principle was unknown until Reynolds derived his well-known Reynolds equation (Reynolds, 1886). Under a number of assumptions, the Reynolds equation provided a basic relationship between the pressure of a convergent oil film in an eccentric bearing with the bearing parameters such as the velocity, geometry and lubricant viscosity. Since then, extensive investigations have been carried out in this field by many researchers trying to obtain the pressure from the Reynolds equation.

A significant step in the bearing theory was developed by Sommerfeld (1904), who developed the Sommerfeld solution of the Reynolds equation for an infinitely long journal bearing. By ignoring the axial gradient of pressure profile and assuming the oil was continuous around the 360 degrees bearing, he succeeded in obtaining a complete analytical solution for the Reynolds equation. However, his sine-wave pressure profile around the whole bearing was later discovered not to be in agreement with experimental evidence because normal fluids cannot withstand large and continuous negative pressure without rupture. In another study, Kingsbury (1931) simulated the pressure distribution of a two-dimensional journal bearing in an electrolytic tank. However, the accuracy of the method was not high because it was restricted by the difficulty in balancing the current inputs to all nodes and poor precision of the components used in the simulation network. McKee and McKee (1932) experimentally determined the pressure distribution in whole surface of a vertically installed bearing. From their 9 sets of tests with different clearances, loads, speeds and viscosities, they found that the positive pressure was only distributed in a film less than π radians circumferentially. Many other investigations have also been carried out on the same subject (Cameron and Wood, 1949; Raimondi and Boyd, 1958a; Raimondi and Boyd, 1958b; Raimondi and Boyd, 1958c; Christopherson, 1941; Li et al., 1977; Hays, 1959).

With the advent of high-speed digital computers, numerical methods are quicker when solving the Reynolds equation. The finite element method has been used prominently to analyse continuum and field problems (Zienkiewicz, 1970). A theoretical study based on the finite element method was carried out by Gethin and Deihi (1987) to investigate the performance of a twin-axial groove cylindrical bore bearing. Basri and Gethin (1990) worked on the finite element method for the solution of hydrodynamic lubrication problems. Sep (2005) investigated the distributions of pressure, temperature and oil flow velocity of a journal bearing with a two-component surface layer. Oladeinde (2009) carried out a study to develop and apply an efficient finite element based simulation of hydrodynamic lubricated bearings with non-Newtonian lubricants and velocity slip. Mane and Soni (2013) presented a paper on the 3D model of hydrodynamic plain journal bearing using COMSOL Multiphysics 4.3a software.

In all the literatures reviewed so far, there was no general analytic solution to the classical Reynolds equation that models the effect of side or end leakage (Budynas-Nisbett, 2008). However, approximate solutions have been obtained by using electrical analysis, mathematical summations, relaxation methods and numerical and graphical methods. This study is presented to proffer solution to the pressure distribution in a journal bearing under hydrodynamic lubrication assuming that the bearing in infinitely short.

2. METHODOLOGY

2.1. Governing Equation and Boundary Conditions

The governing Equation for pressure variation in an infinitely short hydrodynamic journal bearing is shown thus:

$$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial z}\right)=6U\frac{\partial h}{\partial x} \tag{1}$$

Where P = pressure, z = Axial direction, h = Film thickness, μ = Lubricant viscosity,
x = Circumferential direction, U = Linear velocity

Equation (1) is the well-known Reynolds equation for a bearing that is assumed to be infinitely short. This Equation is subject to the following set of initial and boundary conditions.

$$P(\theta,0)=P(\theta,L)=0 \tag{2}$$

Where θ = Angular displacement, L = Length of the bearing
For the purpose of symmetry the following condition was set:

$$\left.\frac{\partial P}{\partial z}\right|_{z=0}=0 \tag{3}$$

The domain of the problem consists of all points between z=0 and *z=L* i.e. Ω=(*0,L*). The domain was divided into a set of line elements, a typical element being of length h_e and located between two arbitrary points A and B. The collection of such elements is called the finite element mesh of the domain. The reason for dividing the domain into finite elements was to represent the geometry of the domain and to approximate the solution over the entire domain.

2.2. Mathematical Analysis

In the development of the weak form, a liner mesh was assumed and placed over the domain. This was done by multiplying Equation (1) by the weighted function (w) and integrating the final Equation over the domain. This results in the mathematical expression in Equation (4).

$$\int_{z_A}^{z_B}\frac{h^3}{\mu}\frac{\partial w}{\partial z}\frac{\partial P}{\partial z}dz+\int_{z_A}^{z_B}6wU\frac{\partial h}{\partial x}dz-wQ_A-wQ_B=0 \tag{4}$$

Equation (4) is known as the weak form of the governing Equation for hydrodynamic lubrication in journal bearing. The weak form requires that the approximation chosen for *P* should be at least linear in *z* so that there are no terms in Equation 4 that are identically zero. Since the primary variable is simply the function itself, the Lagrange family of interpolation functions is admissible. We proposed that *P* is the approximation over a typical finite element domain by the expression:

$$P^e=\sum_{j=1}^{n}P_j^e\psi_j^e(z) \text{ and } w=\psi_i^e(z) \quad i,j=1,2,3 \tag{5}$$

Where $w=\psi_i^e(z)$ is the trial function
In Galerkin's weighted residual method, the weighting functions are chosen to be identical to the trial functions (Reddy, 1993).

On substituting Equation (5) into Equation (4), the following results:

$$\frac{h^3}{\mu}\left[K_{ij}^e\right]\left\{P_j^e\right\} = -6U\frac{\partial h}{\partial x}\left\{F_i^e\right\} + \left\{Q_i^e\right\} \tag{6}$$

Where

$$K_{ij}^e = \int_{z_A}^{z_B} \frac{\partial \psi_i^e(z)}{\partial z}\frac{\partial \psi_j^e(z)}{\partial z}dz \tag{7}$$

$$F_i^e = \int_{z_A}^{z_B} \psi_i^e(z)dz \tag{8}$$

$$Q_i^e = \psi_i^e(z)Q_A + \psi_i^e(z)Q_B \tag{9}$$

Equation (6) is referred to as the finite element based model, Equation (7) is known as the bearing matrix and Equation (8) is referred to as the wedge matrix. Hence, the one-dimensional Lagrange quadratic interpolation function for Equation becomes:

$$\psi_1 = \left(1 - \frac{z}{h_e}\right)\left(1 - \frac{2z}{h_e}\right) \tag{10}$$

$$\psi_2 = \frac{4z}{h_e}\left(1 - \frac{z}{h_e}\right) \tag{11}$$

$$\psi_3 = -\frac{z}{h_e}\left(1 - \frac{2z}{h_e}\right) \tag{12}$$

Where h_e = Elemental length of the bearing

2.2.1. Evaluating the bearing matrix $\lfloor K_{ij} \rfloor$ and wedge matrix $\left\{F^e\right\}$ for journal bearing

To evaluate the K_{ij} matrix, Equations (10)-(12) were substituted accordingly into Equations (7) and (8) respectively to obtain Equations (13) and (14).

$$K^e = \frac{1}{3h_e^3}\begin{bmatrix} 7h_e^2 - 24h_e z_A + 48z_e^2 & -8\left(h_e^2 - 3h_e z_A + 12z_A^2\right) & h_e^2 + 48z_A^2 \\ -8\left(h_e^2 - 3h_e z_A + 12z^2\right) & 16\left(h_e^2 + 12z_A^2\right) & -8\left(h_e^2 + 3h_e z_A + 12z_A^2\right) \\ h_e^2 + 48z_A^2 & -8\left(h_e^2 + 3h_e z_A + 12z_A^2\right) & 7h_e^2 + 24h_e z_A + 48z_e^2 \end{bmatrix} \quad 13)$$

$$F = \begin{Bmatrix} \frac{h_e}{6} - z_A + \frac{2z_A^2}{h_e} \\ \frac{2\left(h_e^2 - 6z_A^2\right)}{3h_e} \\ \frac{h_e}{6} + z_A + \frac{2z_A^2}{h_e} \end{Bmatrix} \tag{14}$$

Equation (13) represents the generalized form of the bearing matrix for the entire domain of the bearing and Equation (14) it represents the generalized form of the wedge matrix for the entire domain of the bearing. In the analysis, the bearing domain was divided into four quadratic elements as shown in Figure 1. Analysing the elements part by part and finally assembling, resulted in the following:

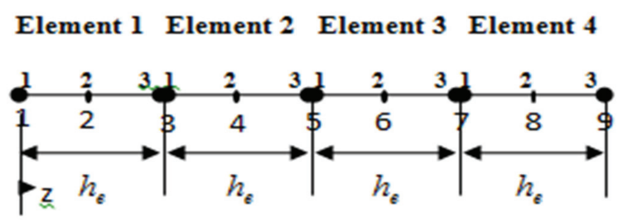

Figure 1: A four quadratic element mesh

$$[K^e]=\frac{1}{3h_e}\begin{bmatrix} 7 & -8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8 & 16 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -8 & 38 & -80 & 49 & 0 & 0 & 0 & 0 \\ 0 & 0 & -80 & 208 & -128 & 0 & 0 & 0 & 0 \\ 0 & 0 & 49 & -128 & 230 & -344 & 193 & 0 & 0 \\ 0 & 0 & 0 & 0 & -344 & 784 & -440 & 0 & 0 \\ 0 & 0 & 0 & 0 & 193 & -440 & 614 & -800 & 433 \\ 0 & 0 & 0 & 0 & 0 & 0 & -800 & 1744 & -944 \\ 0 & 0 & 0 & 0 & 0 & 0 & 433 & -944 & 511 \end{bmatrix} \tag{15}$$

$$\{F^e\}=\frac{h_e}{6}\begin{Bmatrix} 1 \\ 4 \\ 8 \\ -20 \\ 56 \\ -92 \\ 152 \\ -212 \\ 127 \end{Bmatrix} \tag{16}$$

Substituting Equations (15) and (16) in Equation 6, and combining with the boundary conditions in Equations (2) and (3) as well as the parameters listed hereunder results in the following:

$h_e = L/4$, $U = \omega r$, $x = r\theta$ and $h = c(1+\varepsilon\cos\theta)$, we have:

$$\begin{bmatrix} 16 & -8 & 0 & 0 & 0 & 0 & 0 \\ -8 & 38 & -80 & 49 & 0 & 0 & 0 \\ 1 & -80 & 208 & -128 & 0 & 0 & 0 \\ 0 & 49 & -128 & 230 & -344 & 193 & 0 \\ 0 & 0 & 0 & -344 & 784 & -440 & 0 \\ 0 & 0 & 0 & 193 & -440 & 614 & -800 \\ 0 & 0 & 0 & 0 & 0 & -800 & 1744 \end{bmatrix} \begin{Bmatrix} P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{Bmatrix} = \frac{3\omega L^2 \mu \varepsilon \sin\theta}{16c^2(1+\varepsilon\cos\theta)^3} \begin{Bmatrix} 4 \\ 8 \\ -20 \\ 56 \\ -92 \\ 152 \\ -212 \end{Bmatrix} \quad 17)$$

Where L = Length of Bearing, U = Linear velocity, $\omega =$ Angular velocity, r = radius of bearing, c = radial clearance, $\varepsilon =$ eccentricity ratio.

Table 1 contains the values of the parameters that were used for solving the equations.

Table 1: Parameters for calculation for short journal bearing (Mane and Soni, 2013)

Parameters	Short bearing
Lengths of bearing	$2.50\text{x}10^{-2}$ (m)
Diameter of journal	0.05 (m)
L/D ratio	0.50
Radial clearance	$2.50\text{x}10^{-5}$ (m)
Eccentricity	$1.25\text{x}10^{-5}$ (m)
Eccentricity ratio	0.50
Speed of journal	1000 (rpm)
Dynamic viscosity of oil	0.19 (Pa.S)
Inlet temperature	315 (K)

3. RESULTS AND DISCUSSION

3.1. Variation of Pressure with Angular Displacements for Journal Bearing

It is paramount to know how the pressure of the lubricant inside the journal bearing varies with the angular displacements. The results obtained as represented in Figure 2 shows the variation of pressure with angular displacements for a journal bearing which was assumed to be infinitely short. It was assumed that there was no side leakage. The nodal values were the pressures at those points on the bearing. The graph shown was in a sinusoidal wave form. P1 to P9 shows the pressure distribution at the various nodes from 1 to 9 respectively. A combination of both the Neumann and Dirichlet boundary conditions was used. From the boundary conditions used, the pressure increases from the ambient pressure which was taken to be zero at an angular displacement of 0^o and increases significantly till 145.37^o where the pressure becomes maximum with 1.9955 MPa. Thereafter, it began to drop until it gets to 180^o where the pressure became the same as the ambient pressure. From this point onward, we began to experience negative pressure. The negative pressures in this regard were those that were below the ambient pressure. At this point, cavity began to set in. This pressure increases in the negative direction till 210.62^0 and back again to the ambient pressure at 360^o. Then, another cycle began between 360^o and 720^o and so on.

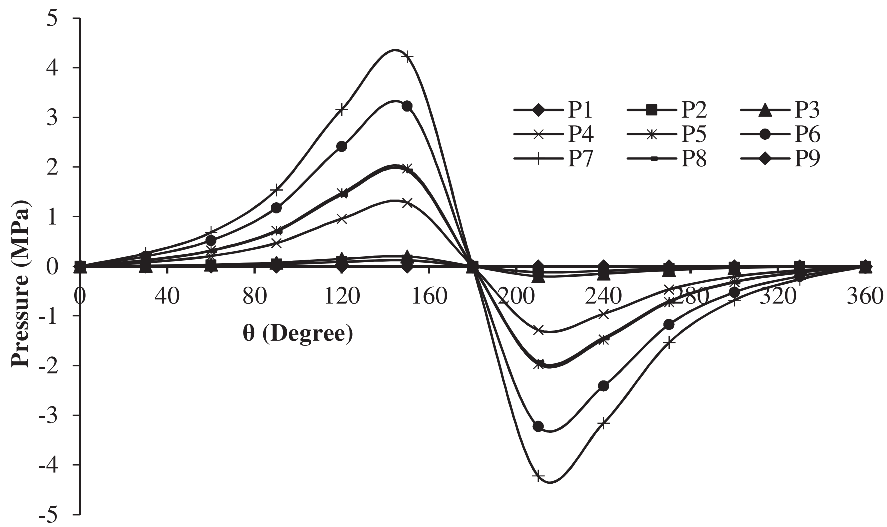

Figure 2: Effect of angular displacement on pressure for short journal bearing

The result from FEM was validated by comparing it with the result from the analytical solution and the result published by Mane and Soni (2013). The comparison shows a strong positive correlation. The FEM approximates the solution to the analytical solution better than the result by Mane and Soni (2013). This is shown in Table 2.

Table 2: Comparison between FEM, COSMOL and Analytical solution

Q (Degrees)	0	30	60	90	120	150	180	210	240	270	300	330	360
Analytical	0	122565	317565	718245	1491345	2003459	0	-2003459	-1491345	-718245	-317565	-121565	0
COSMOL	0	120231	315456	717546	1415657	1954563	0	-1954563	-1415657	-717546	-315456	-120231	0
FEM	0	122000	318375	718023	1473959	1969648	0	-1969648	-1473959	-718023	-318375	-122000	0

3.2. Variation of Pressure along the Axial Direction for Journal Bearing

It was observed that at the beginning of the bearing where the pressure was equal to the ambient pressure, the pressure increases thereafter till about 0.01875 m where the pressure was at maximum. From this point to the other trailing end of the bearing, the pressure decreases to the ambient pressure at L = 0.025 m. This was shown in Figure 3. At the early stage in the bearing, the pressure increase was seen not to be that significant. This was usually between the ranges of $0 \leq L \leq 0.00625$ m. From this point onward, the pressure begins to increase significantly. The reverse was the case when the bearing's angular displacement was in the range of $180^o \leq \theta \leq 360^o$.

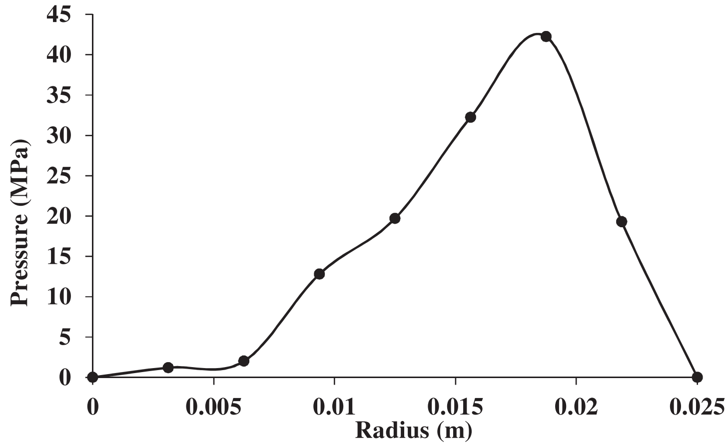

Figure 3: Variation of pressure as a function of axial displacement for short journal bearing

4. CONCLUSION

The pressure distribution of the hydrodynamic journal bearing lubricated with oil under steady state and constant temperature consideration has been analysed. Based on the results obtained, the following conclusions can be reached.

1. The finite element method (FEM) form of the general governing Reynolds equation was derived and implemented for hydrodynamic journal bearing.
2. Using Reynolds equation, a numerical solution has been developed for short journal bearing under steady state.
3. The finite element method provides the node by node results for pressure distribution.
4. Using FEM, the result obtained was compared with COSMOL and analytical solution. It was found that FEM gives approximately identical solution for short journal bearing.

5. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

REFERENCES

Basri, S. and Gethin, D.T. (1990). A Comparative Study of the Thermal Behaviour of Profile Bore Bearings. Tribology International, 4(12), pp. 265-276.

Budynas−Nisbett, (2008). Shigley's Mechanical Engineering Design, 8th Ed. McGraw-Hill.

Cameron, A. and Wood, W.L. (1949). The full journal bearing. *Proceedings of the Institution of Mechanical Engineers*, 161(1), pp. 59-72.

Christopherson, D.G. (1941). A new mathematical method for the solution of film lubrication problems. *Proceedings of the Institution of Mechanical Engineers*, 146(1), pp. 126-135.

Gethin, D.T. and EI Deihi, M.K.I. (1987). Effect of Loading Direction on the Performance of a Twin Axial Groove Cylindrical Bore Bearing. *Tribology International*, 20(3), pp. 179-185.

Hays, D.F. (1959). A variational approach to lubrication problems and solution of the finite journal bearings. *Journal of Basic Engineering, Transaction of the American Society of Mechanical Engineers, Series D*, 81, pp. 13-23.

Kingsbury, A. (1931). On Problems in the Theory of Fluid-Film Lubrication, With an Experimental Method of Solution, *Transaction of the American Society of Mechanical Engineers*, 53, pp. 59-75.

Li, D.F., Allaire, P.E. and Barrett, L.E. (1977). Analytical Dynamics of Partial journal bearings with applications, Association for the Study of Literature and Environment /American Society of Mechanical Engineers, Lubrication conference, Kansas City, Missouri.

Mane, R.M. and Soni, S. (2013). Analysis of Hydrodynamic Plain Journal Bearing. In: *Excerpt from the proceedings of the 2013 COSMOL conference in Bangalore*.

McKee, S.A. and McKee, T.R. (1932). Pressure distribution in oil films of journal bearings. *Transaction of the American Society of Mechanical Engineers*, 57, pp. 149-165.

Raimondi, A.A. and Boyd J. (1958a). A solution for the finite journal bearing and its application to analysis and design: I, *Transaction of the Association for the Study of Literature and Environment*, 1(1), pp. 159-174.

Raimondi, A.A. and Boyd J. (1958b). A solution for the finite journal bearing and its application to analysis and design: II, *Transaction of the Association for the Study of Literature and Environment*, 1(1), pp. 175-193.

Raimondi, A.A., and Boyd J. (1958c). A solution for the finite journal bearing and its application to analysis and design: III, *Transaction of the Association for the Study of Literature and Environment*, 1(1), pp. 194-203.

Reddy, J.N. (1993). Introduction to the Finite Element Method, 2nd Ed. McGraw-Hill series in Mechanical Engineering.

Reynolds, O. (1886). On the Theory of Lubrication and Its Application to Mr. Beauchamp Tower's Experiments, Including an Experimental Determination of the Viscosity of Olive Oil. *Proceedings of the Royal Society of London*, 40(242-245), pp. 191-203.

Sep, J. (2005). Three dimensional hydrodynamic analysis of a journal bearing with a two component surface layer. *Tribology International*, 38, pp. 97-104.

Sommerfeld A. (1904). Zur hydrodynamische theorie der schmeirmittle-beibung. *Zeitschrift furr Mathematik und Physik*, 50, pp. 97.

Zienkiewicz, O.C. (1970). The Finite Element Method, 2nd Ed. McGraw-Hill.