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Original Research Article

EFFECT OF ECCENTRICITY RATIO AND ROTATIONAL SPEED ON THE PRESSURE VARIATION OF FLUID IN HYDRODYNAMIC JOURNAL BEARING

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ABSTRACT

In this work, an analysis has been carried to determine the effect of eccentricity ratio and rotational speed on the pressure variation of fluid (lubricant) in a hydrodynamic journal bearing. The classical Reynolds equation was used to represent the pressure behaviour in a journal bearing assumed to be infinitely short. This study was carried out using the finite element method (FEM). The domain of the bearing was discretized and pressure analysis was carried out on each of these finite elements. The overall results from these finite elements were finally assembled to represent the pressure distribution in the entire domain of the bearing. The result obtained shows that as the eccentricity ratio increases, the pressure in the bearing also increases. This was shown for different values of eccentricity between 0.1 and 0.8. The same relationship was obtained for the angular speed. It was shown that the pressure was directly proportional to the rotational speed and this was shown for different rotational speed between 1000 rad/s and 10000 rad/s. The result obtained from this study shows to have a strong positive correlation with existing result in literature.

1. INTRODUCTION

Journal bearings have been widely used for a long time but their lubrication principle was unknown until Reynolds derived his well-known Reynolds equation (Reynolds, 1886). Under a number of assumptions, the Reynolds equation gave a basic relationship of the pressure of a convergent oil film in an eccentric bearing with the bearing parameters, such as the velocity, geometry and lubricant viscosity. Since then, extensive investigations have been carried out in this field by many researchers trying to obtain the pressure from the Reynolds equation.

A significant step in the bearing theory was developed by Sommerfeld (1904), who developed the Sommerfeld solution of Reynolds equation for an infinitely long journal bearing. By ignoring the axial gradient of pressure profile and assuming the oil was continuous around the 360 degrees bearing, he succeeded in obtaining a complete analytical solution for the Reynolds equation. Kingsbury (1931) simulated the pressure distribution of a two-dimensional journal bearing in an electrolytic tank. However, the accuracy of the method was not high because it was restricted by the difficulty in balancing the current inputs to all nodes and poor precision of the components used in the simulation network. McKee and McKee (1932) experimentally determined the pressure distribution in whole surface of the vertically installed bearing. From their 9 sets of tests with different clearances, loads, speeds and viscosities, they found that the positive pressure was only distributed in a film less than π radian circumferentially. The bell-shaped circumferential distribution and parabola-shaped axial distribution are very close to today's bearing theory.

With the advent of high-speed digital computers, numerical methods are quicker when solving the Reynolds equation. The finite element method has been used prominently to analyse continuum and field problems (Zienkiewicz, 1970). Stephenson (1997) developed Bearing Design System, a program to perform bearing analysis using the finite element method. Booker and Huebner (1972) applied the finite element method to the general lubrication problem with a systematic description of procedures. Huebner (1975) provided the most recent and extensive treatment of fluid film lubrication. A theoretical investigation was carried out by Gethin and Medwell (1984) using the finite element method for a high speed bearing. A theoretical study based on the finite element method was carried out by Gethin and Deihi (1987) to investigate the performance of a twin-axial groove cylindrical bore bearing. Basri and Gethin (1990) worked on the finite element method for the solution of hydrodynamic lubrication problems. Sep (2005) investigated the distributions of pressure, temperature and oil flow velocity of a journal bearing with a two-component surface layer. Mane and Soni (2013) presented a 3D model of hydrodynamic plain journal bearing using COMSOL Multiphysics 4.3a software. From the model, pressure distribution in plain journal bearing was obtained by steady state analysis of plain journal bearing using the generalized Reynolds Equation by applying the Sommerfeld boundary conditions.

In all the literatures reviewed so far, there is no general analytic solution to the classical Reynolds equation that models the effect of side or end leakage (Budynas-Nisbett, 2008). However, approximate solutions have been obtained by using electrical analysis, mathematical summations, relaxation methods and graphical methods. This study is presented to analyse the effects of eccentricity ratio and rotational speed on the pressure distribution in a journal bearing under hydrodynamic lubrication assuming that the bearing is infinitely short.

2. METHODOLOGY

2.1. Governing Equation and Boundary Conditions

The governing Equation for pressure variation in an infinitely short hydrodynamic journal bearing is shown thus:

$$\frac{\partial}{\partial z}\left(\frac{h^3}{\mu}\frac{\partial P}{\partial z}\right) = 6U\frac{\partial h}{\partial x} \tag{1}$$

Where P = pressure, z = axial direction, h = film thickness, μ= lubricant viscosity, x = circumferential direction, U = linear velocity

Equation (1) is the well-known Reynolds equation for a bearing that is assumed to be infinitely short. This Equation is subject to the following set of initial and boundary conditions.

$$P(\theta,0) = P(\theta,L) = 0 \tag{2}$$

Where θ= Angular displacement, L = Length of the bearing

For symmetry reasons, the following condition was set:

$$\left.\frac{\partial P}{\partial z}\right|_{z=0} = 0 \tag{3}$$

The domain of the problem consists of all points between z=0 and $z=L$ i.e. $\Omega=(0,L)$. The domain was divided into a set of line elements, a typical element being of length h_e and located between two arbitrary points A and B. The collection of such elements is called the finite element mesh of the domain. The reason for dividing the domain into finite elements was to represent the geometry of the domain and to approximate the solution over the entire domain.

2.2. Mathematical Analysis

In the development of the weak form, we assumed a linear mesh and placed it over the domain. This was done by multiplying Equation (1) by the weighted function (w) and integrating the final Equation over the domain. This results in the mathematical expression in Equation (4).

$$\int_{z_A}^{z_B}\frac{h^3}{\mu}\frac{\partial w}{\partial z}\frac{\partial P}{\partial z}dz + \int_{z_A}^{z_B}6wU\frac{\partial h}{\partial x}dz - wQ_A - wQ_B = 0 \tag{4}$$

Equation (4) is known as the weak form of the governing Equation for hydrodynamic lubrication in journal bearing. The weak form requires that the approximation chosen for P should be at least linear in z so that there are no terms in Equation (4) that are identically zero. Since the primary variable is simply the function itself, the Lagrange family of interpolation functions is admissible. We proposed that P is the approximation over a typical finite element domain by the expression:

$$P^e = \sum_{j=1}^{n} P_j^e \psi_j^e(z) \text{ and } w = \psi_i^e(z) \quad i,j = 1,2,3 \tag{5}$$

where $w = \psi_i^e(z)$ is the trial function

In Galerkin's weighted residual method, the weighting functions are chosen to be identical to the trial functions (Reddy, 1993). Substituting Equation (5) into Equation (4) results in:

$$\frac{h^3}{\mu}\left[K_{ij}^e\right]\left\{P_j^e\right\} = -6U\frac{\partial h}{\partial x}\left\{F_i^e\right\} + \left\{Q_i^e\right\} \tag{6}$$

Where

$$K_{ij}^e = \int_{z_A}^{z_B} \frac{\partial \psi_i^e(z)}{\partial z}\frac{\partial \psi_j^e(z)}{\partial z}dz \tag{7}$$

$$F_i^e = \int_{z_A}^{z_B} \psi_i^e(z)dz \tag{8}$$

$$Q_i^e = \psi_i^e(z)Q_A + \psi_i^e(z)Q_B \tag{9}$$

Equation (6) is referred to as the finite element based model, Equation (7) is known as the bearing matrix and Equation (8) is referred to as the wedge matrix. Hence, the one-dimensional Lagrange quadratic interpolation function for Equation becomes:

$$\psi_1 = \left(1 - \frac{z}{h_e}\right)\left(1 - \frac{2z}{h_e}\right) \tag{10}$$

$$\psi_2 = \frac{4z}{h_e}\left(1 - \frac{z}{h_e}\right) \tag{11}$$

$$\psi_3 = -\frac{z}{h_e}\left(1 - \frac{2z}{h_e}\right) \tag{12}$$

2.2.1. Evaluating the bearing matrix $\lfloor K_{ij} \rfloor$ and wedge matrix $\{F^e\}$ for journal bearing

To evaluate the K_{ij} matrix, Equations (10)-(12) were substituted accordingly into Equations (7) and (8) respectively to obtain Equations (13) and (14).

$$K^e = \frac{1}{3h_e^3}\begin{bmatrix} 7h_e^2 - 24h_e z_A + 48z_e^2 & -8\left(h_e^2 - 3h_e z_A + 12z_A^2\right) & h_e^2 + 48z_A^2 \\ -8\left(h_e^2 - 3h_e z_A + 12z^2\right) & 16\left(h_e^2 + 12z_A^2\right) & -8\left(h_e^2 + 3h_e z_A + 12z_A^2\right) \\ h_e^2 + 48z_A^2 & -8\left(h_e^2 + 3h_e z_A + 12z_A^2\right) & 7h_e^2 + 24h_e z_A + 48z_e^2 \end{bmatrix} \tag{13}$$

$$F = \begin{Bmatrix} \dfrac{h_e}{6} - z_A + \dfrac{2z_A^2}{h_e} \\ \dfrac{2\left(h_e^2 - 6z_A^2\right)}{3h_e} \\ \dfrac{h_e}{6} + z_A + \dfrac{2z_A^2}{h_e} \end{Bmatrix} \tag{14}$$

Equation (13) represents the generalized form of the bearing matrix for the entire domain of the bearing and Equation (14) it represents the generalized form of the wedge matrix for the entire domain of the bearing. In the analysis, the bearing domain was divided into four quadratic elements as shown in Figure 1. Analysing the elements part by part and finally assembling, resulted in the following:

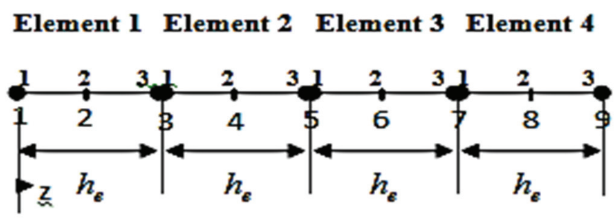

Figure 1: A four quadratic element mesh

$$\left[K^e\right] = \frac{1}{3h_e}\begin{bmatrix} 7 & -8 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -8 & 16 & -8 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -8 & 38 & -80 & 49 & 0 & 0 & 0 & 0 \\ 0 & 0 & -80 & 208 & -128 & 0 & 0 & 0 & 0 \\ 0 & 0 & 49 & -128 & 230 & -344 & 193 & 0 & 0 \\ 0 & 0 & 0 & 0 & -344 & 784 & -440 & 0 & 0 \\ 0 & 0 & 0 & 0 & 193 & -440 & 614 & -800 & 433 \\ 0 & 0 & 0 & 0 & 0 & 0 & -800 & 1744 & -944 \\ 0 & 0 & 0 & 0 & 0 & 0 & 433 & -944 & 511 \end{bmatrix} \tag{15}$$

$$\left\{F^e\right\} = \frac{h_e}{6}\begin{Bmatrix} 1 \\ 4 \\ 8 \\ -20 \\ 56 \\ -92 \\ 152 \\ -212 \\ 127 \end{Bmatrix} \tag{16}$$

Substituting Equations (15) and (16) in Equation 6, and combining with the boundary conditions in Equations (2) and (3) as well as the parameters listed hereunder results in the following:

$h_e = L/4$, $U = \omega r$, $x = r\theta$ and $h = c(1+\varepsilon\cos\theta)$, we have:

$$\begin{bmatrix} 16 & -8 & 0 & 0 & 0 & 0 & 0 \\ -8 & 38 & -80 & 49 & 0 & 0 & 0 \\ 1 & -80 & 208 & -128 & 0 & 0 & 0 \\ 0 & 49 & -128 & 230 & -344 & 193 & 0 \\ 0 & 0 & 0 & -344 & 784 & -440 & 0 \\ 0 & 0 & 0 & 193 & -440 & 614 & -800 \\ 0 & 0 & 0 & 0 & 0 & -800 & 1744 \end{bmatrix} \begin{Bmatrix} P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{Bmatrix} = \frac{3\omega L^2 \mu\varepsilon\sin\theta}{16c^2(1+\varepsilon\cos\theta)^3} \begin{Bmatrix} 4 \\ 8 \\ -20 \\ 56 \\ -92 \\ 152 \\ -212 \end{Bmatrix} \quad (17)$$

Where L = Length of Bearing, U = Linear velocity, ω = Angular velocity, r = radius of bearing, c = radial clearance, ε = eccentricity ratio.

Table 1 contains the values of the parameters that were used for solving the equations.

Table 1: Parameters for calculation for short journal bearing (Mane and Soni, 2013)

Parameters	Short bearing
Lengths of bearing	$2.50\text{x}10^{-2}$ (m)
Diameter of journal	0.05 (m)
L/D ratio	0.50
Radial clearance	$2.50\text{x}10^{-5}$ (m)
Eccentricity	$1.25\text{x}10^{-5}$ (m)
Eccentricity ratio	0.50
Speed of journal	1000 (rpm)
Dynamic viscosity of oil	0.19 (Pa.S)
Inlet temperature	315 (K)

3. RESULTS AND DISCUSSION

3.1. Variation of Pressure with Eccentricity Ratio for Journal Bearing

The results obtained from the relationship between the pressure distribution in the bearing and the angular displacement for a journal bearing whose L/D ratio was 0.5 shows that the pressure variations were critical at the 145.36^{o} angular displacement. Therefore it was necessary to know the variation of pressure with eccentricity ratio at 145.36^{o}. The result obtained from this study is shown in Figure 2. It was observed that as the eccentricity ratio increased, the pressure in the bearing also increased. Also, the negative pressure increased with an increase in eccentricity ratio. This increase in pressure is as a result of the decrease in the gap between the journal and the bearing. The lubricant will now have to flow through this gap at a higher pressure. Also, as the value of eccentricity ratio increases, the steepness of the plot of pressure variation against angular displacement increases. This is shown in Figure 3. The steepness is a function of the increase in the pressure. At the point where e = 1, the bearing is now between hydrodynamic and boundary dominated lubrication. The bearing characteristic at this point will now be different. To test for the accuracy of the results, a percentage error computation was carried out between this work and the result published by Mane (2013) using COSMOL. This is shown in Table 2. The comparison shows a strong positive correlation between the two results.

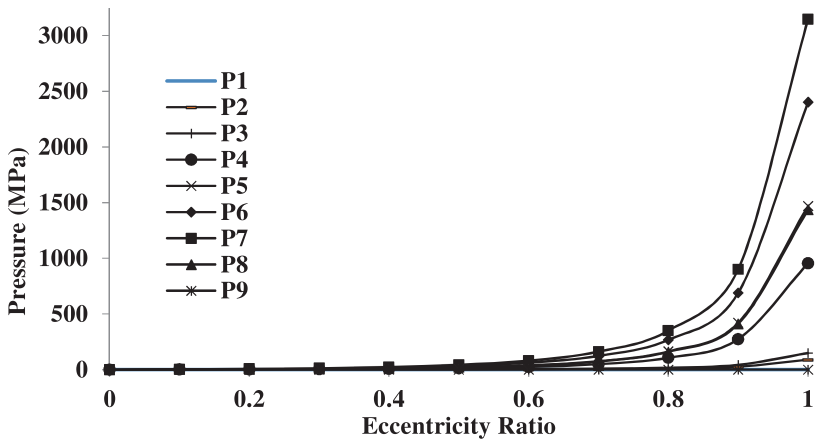

Figure 2: Effect of eccentricity ratio on pressure for short journal bearing

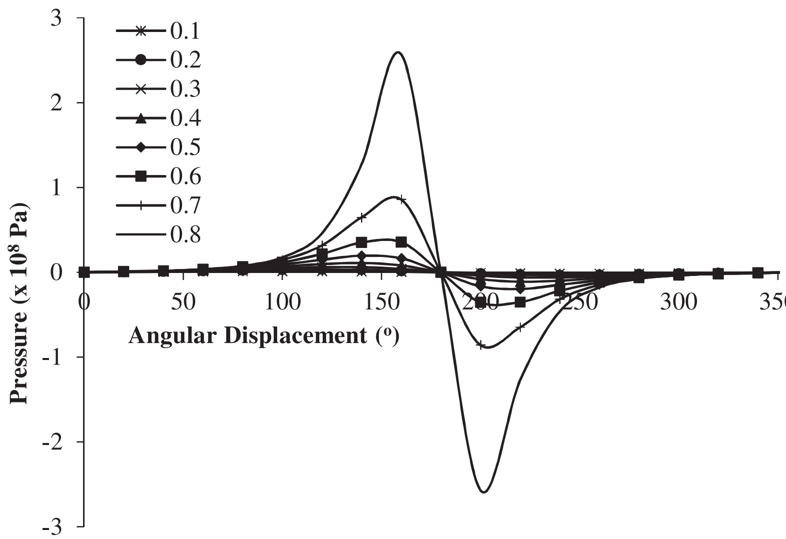

Figure 3: Effect of angular displacement on pressure at different eccentricity ratios

Table 2: Percentage error between COSMOL solution and the results of this work

Eccentricity Ratio	Percentage errors									
	0^0	20^0	40^0	60^0	80^0	100^0	120^0	140^0	160^0	180^0
0.1	0.0000	0.7863	0.1859	2.7359	0.6061	0.1418	0.4119	0.0852	0.0151	0.0000
0.2	0.0000	0.7948	0.7966	0.3675	0.0910	0.1938	0.2922	0.0329	0.0545	0.0000
0.3	0.0000	0.0848	0.7757	0.9440	0.2208	0.2003	0.1643	0.0165	0.0251	0.0000
0.4	0.0000	0.2091	0.7005	0.3785	0.6834	0.0285	0.0103	0.0903	0.0247	0.0000
0.5	0.0000	0.6106	0.5551	0.6144	0.2351	0.1613	0.6739	0.0509	0.6031	0.0000
0.6	0.0000	0.1688	0.5285	0.2069	0.4282	0.8401	0.0459	0.0028	0.2808	0.0000
0.7	0.0000	0.3261	1.0120	1.2596	0.3261	0.4770	0.0032	0.0154	0.0233	0.0000
0.8	0.0000	1.0869	0.7469	0.0169	0.0380	0.5611	0.0217	0.0785	0.0778	0.0000

Eccentricity Ratio	Percentage errors								
	200^0	220^0	240^0	260^0	280^0	300^0	320^0	340^0	360^0
0.1	0.0151	0.0852	0.4119	0.1418	0.6061	2.7359	0.1859	0.7863	0.0000
0.2	0.0545	0.0329	0.2922	0.1938	0.0910	0.3675	0.7966	0.7948	0.0000
0.3	0.0251	0.0165	0.1643	0.2003	0.2208	0.9440	0.7757	0.0848	0.0000
0.4	0.0247	0.0903	0.0103	0.0285	0.6834	0.3785	0.7005	0.2091	0.0000
0.5	0.6031	0.0509	0.6739	0.1613	0.2351	0.6144	0.5551	0.6106	0.0000
0.6	0.2808	0.0028	0.0459	0.8401	0.4282	0.2069	0.5285	0.1688	0.0000
0.7	0.0233	0.0154	0.0032	0.4770	0.3261	1.2596	1.0120	0.3261	0.0000
0.8	0.0778	0.0785	0.0217	0.5611	0.0380	0.0169	0.7469	1.0869	0.0000

3.2. Variation of Pressure with Angular Speed

The results obtained from the analysis of the pressure variation with the eccentricity ratio shows that the pressure variation was very high at a high eccentricity and such variations were very pronounced at an eccentricity ratio of 0.8. It is therefore necessary to know how the pressure varies at this high eccentricity level with variation of the angular speed. The parameters to be substituted for are the angular displacement and the eccentricity ratio which are 145.36^o and 0.8 respectively. The result obtained show that the change in angular varies linearly with a change in pressure. That is to say that, the higher the angular speed, the higher the pressure. The value of angular velocity used varies from 1000 rad/s to 10000 rad/s. This can be seen from Figure 4. Note that the result displayed is the pressure variation with angular velocity at the centre of the bearing in the axial direction.

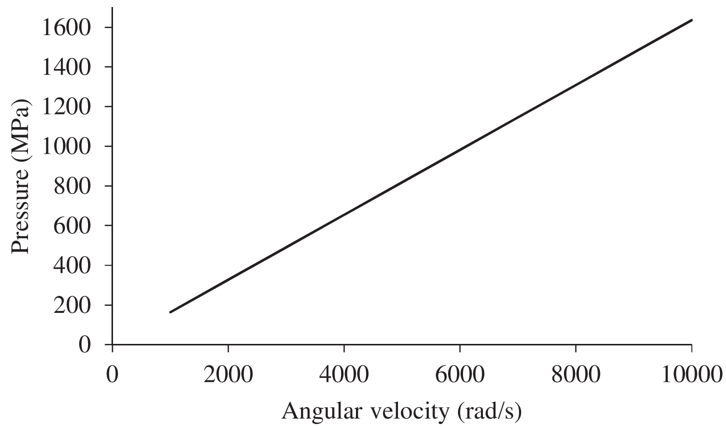

Figure 4: Variation of pressure with angular speed for short journal bearing

4. CONCLUSION

The effects of eccentricity ratio and rotational speed on the pressure distribution in hydrodynamic journal bearing lubricated with oil under steady state and constant temperature consideration has been analysed. Based on the results and discussion presented in the preceding section, the following conclusions can be made for journal bearing studied. The finite element method (FEM) form of the general governing Reynolds equation was derived and implemented for hydrodynamic journal bearing. Using Reynolds equation analytical model developed for short journal bearing to find steady state characteristics of journal bearing, it was observed that the eccentricity ratio and the rotational speed of the journal were directly proportional to the pressure in the bearing. The finite element method provides the node by node results for pressure distribution. Using FEM, the result obtained was compared with COSMOL solution. It was found that FEM gives approximately identical solution for short journal bearing.

5. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

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