Nigerian Research Journal of Engineering and Environmental Sciences
Journal homepage: www.rjees.com

Original Research Article

INTERPRETATION OF POLE – DIPOLE 2-DIMENSIONAL GEOELECTRICAL RESISTIVITY SURVEY DATA USING BLOCKY AND SMOOTHNESS CONSTRAINED INVERSION METHODS

***Alile, O.M., Aigbogun, C.O., Airen, J.O., Ighodalo, J.E. and Eguavoen, K.**

Department of Physics, Faculty of Physical Sciences, University of Benin, Edo State, Nigeria
*monday.alile@uniben.edu

ARTICLE INFORMATION

Article history:
Received 06 May 2017
Revised 13 May 2017
Accepted 14 May 2017
Available online 01 June 2017

Keywords:
Blocky
Inversion
Smoothness constrained
Resistivity
Tomography

ABSTRACT

The interpretation of 2 – Dimensional geoelectrical resistivity data in the field at Ekiugbo, Uhunmwode LGA, Edo State, Nigeria was carried out using the smoothness constrained and blocky inversion codes. This was aimed at identifying the areas of application of these interpretation techniques. Pole-dipole array method was used for the transverse in acquiring the data. The survey spanned a transverse of 300 m. In order to obtain a 2-D model of the subsurface, the field data were inverted using the RES2DINV software. From the inversion it was observed that the smoothing of model resistivity inversion method is suitable for areas with smooth variation of subsurface resistivity while the blocky inversion method is best suited for areas with sharp interfaces or boundaries between different regions with different resistivity values.

1. INTRODUCTION

The smoothness constrained and blocky inversion codes were adopted in this study with the aim of identifying the appropriate locations in which the interpretation techniques are suitable. To achieve this is to carry out electrical resistivity tomography (ERT) in the field and invert the data. Electrical resistivity tomography (ERT) is a geophysical technique for imaging subsurface structures from electrical measurements made at the surface of the ground. It is currently the most frequently used geoelectrical method (Dahlin 1996; Loke 2010). It is broadly used in geomorphology (Schrott and Sass 2008), engineering (Daily and

Ramirez 2000), archaeology (Cardarelli, 2002) and environmental studies (Dahlin *et al.,* 2006; Amidu and Dunbar, 2008; Georgaki *et al.,* 2008; Sechman *et al.*, 2013). As with all other resistivity methods, ERT employs an artificial source of electric current (DC or low frequency AC), which is introduced into the ground through point metallic stakes (electrodes). The purpose of electrical resistivity surveys is to determine the subsurface resistivity distribution.

Usually only the earth's resistivity or electrical potential data are available, so a successful application of ERT requires a reliable inversion algorithm that can convert the measured data to spatial distribution of resistivity (Zhou, 2007). Geophysical inversion seeks to find a model that gives a response that is similar to the actual measured values. The model is an idealized mathematical representation of a section of the Earth (Loke, 2004). The model has a set of model parameters that are physical quantities to be estimated from the observed data. The model response is the synthetic data that can be calculated from the mathematical relationship defining the model for a given set of model parameters. All inversion methods essentially try to determine a model for the subsurface whose response agrees with the measured data subject to certain restrictions.

The solution of the inverse problem (i.e. inverse modeling) requires the assumption of an initial model. A decision must be made both on the type (e.g. horizontally layered, dipped or tectonic structure) and the quantitative (petrophysical and geometrical) properties of the elements of the model. It is supported, on the one hand, by the a priori geological and geophysical knowledge about the investigated geological structure and, on the other hand, the computational (hardware and software) facilities. The latter is of great importance in calculating the theoretical responses (i.e. calculated data) over complex structures by using proper physical principles and different measurement arrays.

2. BASIC INVERSE THEORY

The mathematical link between the model parameters and the model response for 2-D and 3-D resistivity models is provided by the finite-difference Dey and Morrison, (1979) or finite-element methods (Silvester and Ferrari 1990).

In all optimization methods, an initial model is modified in an iterative manner so that the difference between the model response and the observed data values is reduced. The set of observed data can be written as a column vector **y** given by:

$$y = \text{Col}\,(y_1, y_2, \ldots.., y_m) \tag{1}$$

Where *m* is the number of measurements.

The model response f can be written in a similar form.

$$f = \text{Col}(\, f_1, f_2, \ldots\ldots., f_m\,) \tag{2}$$

For resistivity problems, it is a common practice to use the logarithm of the apparent resistivity values for the observed data and model response, and the logarithm of the model values as the model parameters. The model parameters can be represented by the following vector:

$$Q = \text{Col}(\, q_1, q_2, \ldots\ldots., q_m\,) \tag{3}$$

where m is the number of model parameters. The difference between the observed data and the model response is given by the discrepancy vector g that is defined by:

$$g = y - f \tag{4}$$

In the least-squares optimization method, the initial model is modified such that the sum of squares error (E) of the difference between the model response and the observed data values is minimized.

$$E = g^T g \tag{5}$$

To reduce the above error value, the following Gauss-Newton equation is used to determine the change in the model parameters that should reduce the sum squares error (Lines and Treitel, 1984).

$$J^T J \, \Delta q_i = J^T g \tag{6}$$

Where Δq is the model parameter change vector, and J is the Jacobian matrix (of size *m* by *n*) of partial derivatives. The elements of the Jacobian matrix are given by:

$$J_{ij} = \frac{\partial f_i}{\partial q_j} \tag{7}$$

That is the change in the ith model response due to a change in the jth model parameter. After calculating the parameter change vector, a new model is obtained by:

$$q_{k+1} = q_k + \Delta q_k \tag{8}$$

In practice, the simple least-squares equation (Equation 6) is rarely used by itself in geophysical inversion. In some situations the matrix product $J^T J$ might be singular, and thus the least-squares equation does not have a solution for Δq. Another common problem is that the matrix product $J^T J$ is nearly singular. This can occur if a poor initial model that is very different from the optimum model is used. The parameter change vector calculated using equation 6 can have components that are too large such that the new model calculated with Equation 8 may give values that are not realistic. One common method to avoid this problem is the Marquardt Levenberg modification (Lines and Treitel 1984) to the Gauss- Newton equation that is given by:

$$(J^T J + \lambda I) \, \Delta q_k = J^T g \tag{9}$$

Where I is the identity matrix. The factor λ is known as the Marquardt or damping factor. The damping factor effectively constrains the range of values that the components of parameter change vector (Δq) can take. While the Gauss-Newton method attempts to minimize the sum of squares of the discrepancy vector only, the Marquardt-Levenberg method modification also minimizes a combination of the magnitude of the discrepancy vector and the parameter change vector. This method has been successfully used in the inversion of resistivity sounding data where the model consists of a small number of layers.

However when the number of model parameters is large, such as in 2D and 3D inversion model that consist of a large number of small cells, the model produced by this method can have an erratic resistivity distribution with spurious high or low resistivity zones (Constable et al., 1987). To overcome this problem, the Gauss-Newton least-squares equation is further modified so as to minimize the spatial variations in the model parameters (i.e. the model resistivity values change in a smooth or gradual manner). This smoothness-constrained least-squares method (Ellis and Oldenburg 1994) has the following mathematical form.

$$(J^T J + \lambda I)\,\Delta q_k = J^T g - \lambda F q_k \qquad (10)$$

Where $F = \alpha_X C_X{}^T C_X + \alpha_Y C_Y{}^T C_Y + \alpha_Z C_Z{}^T C_Z$

And Cx, Cy and Cz are the smoothing matrices in the *x*-, *y*- and *z*-directions. α_X, α_Y and α_Z are the relative weights given to the smoothness filters in the *x*-, *y*- and *z*-directions.

Equation 10 also tries to minimize the square of the spatial changes, or roughness, of the model resistivity values. It is in fact an l_2 norm smoothness-constrained optimization method. This tends to produce a model with a smooth variation of resistivity values. This approach is acceptable if the actual subsurface resistivity varies in a smooth and gradational manner. In some cases, the subsurface geology consists of a number of regions that are internally almost homogeneous but with sharp boundaries between different regions. For such cases, the inversion formulation in equation 10 can be modified so that it minimizes the absolute changes in the model resistivity values (Claerbout and Muir, 1973). This can sometimes give significantly better results**.** Technically, this is referred to as an l_1 norm smoothness constrained optimization method, or more commonly known as a blocky inversion method. A number of techniques can be used for such a modification. One simple method to implement an l_1 norm based optimization method using the standard least-squares formulation is the iteratively reweighted least-squares method (Wolke and Schwetlick, 1988). The optimization equation in Equation 10 is modified to:

$$(J^T J + \lambda F_R)\,\Delta q_k = J^T R_d\, g - \lambda F_R\, q_k\,, \qquad (11)$$

with $F = \alpha_X C_X{}^T Rm\, C_X + \alpha_Y C_Y{}^T Rm\, C_Y + \alpha_Z C_Z{}^T Rm\, C_Z$

Where R_d and R_m are weighting matrices introduced so that different elements of the data misfit and model roughness vectors are given equal weights in the inversion process. Equation 11 provides a general method that can be further modified if necessary to include known information about the subsurface geology.

3. METHODOLOGY

3.1. Study Location

The study was carried out at Ekeiugbo village in Uhunmwode Local Government Area, Edo State, Nigeria. Ekiugbo village is by the Benin by–pass along Auchi road. The study location lies between Ebueneki and Idibo in Uhunmwode Local Government Area.

3.2. Research Method

In this study, electrical resistivity survey was carried out using pole – dipole array method to acquire the data and the interpretation was done using both smoothness and blocky inversion methods. The inversion software used for the work is RES2DINV which has inversion options. In inverting the data obtained from the electrical resistivity survey using the pole – dipole array, the default smoothness-constrained inversion option was done after which bad data points were removed and the option was repeated. Thereafter, the smoothing of model resistivity option and the blocky inversion option were used in turn to invert the data after which the inverse model resistivity section from each inversion option was compared.

The inversion method was done to determine a model for the subsurface whose response agrees with the measured data subject to certain restrictions. In the cell-based method used by the RES2DINV program, the model parameters are the resistivity values of the model cells, while the data is the measured apparent resistivity values.

4. RESULTS AND DISCUSSION

The results are presented in Figures 1 and 2 for interpretation using smoothing of model resistivity option and blocky of model resistivity option respectively. The inverse model resistivity sections obtained for the data sets showed variations of resistivity in the subsurface. The default smoothness-constrained option showed a slight variation from that obtained after editing the data sets. The smoothing of model resistivity inversion option gave a clearer view of the smooth variation of the subsurface resistivity as shown in Figure 1.

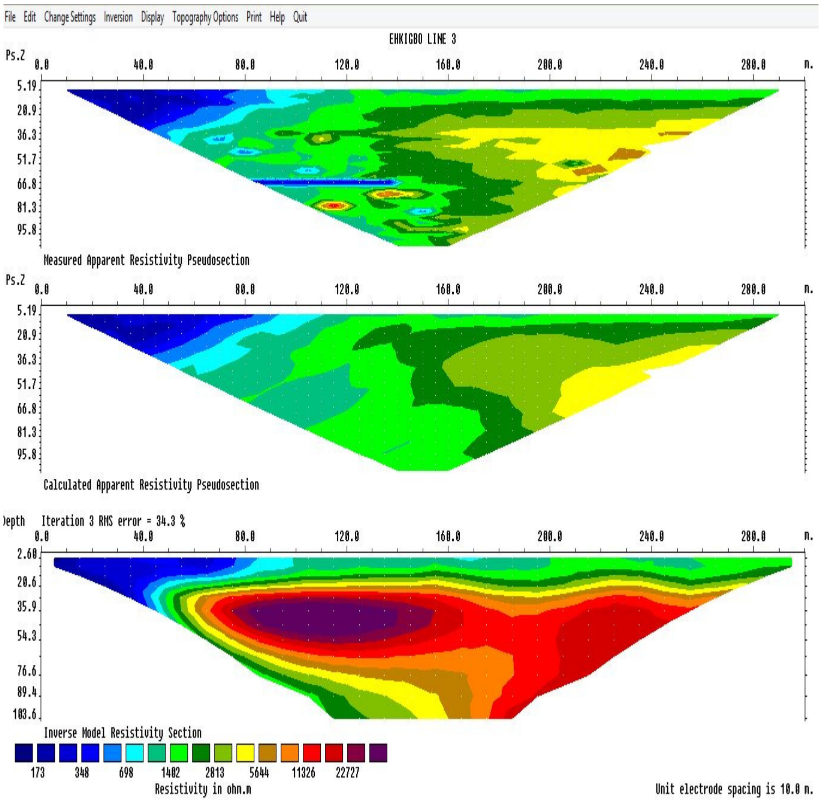

Figure 1: 2D interpretation using smoothing of model resistivity option

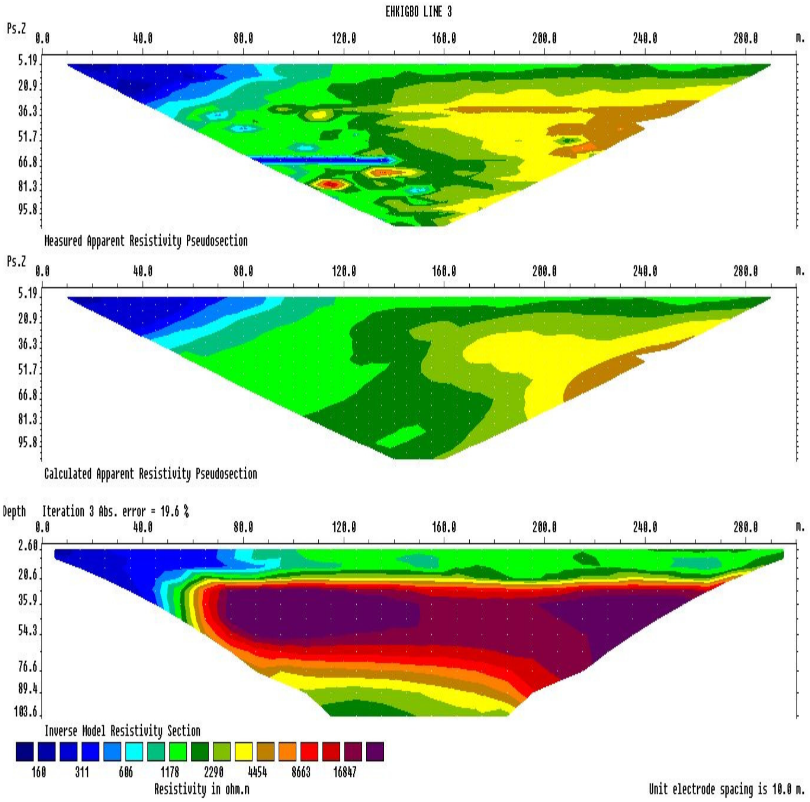

Figure 2: 2D interpretation using blocky of model resistivity option

However, the blocky inversion option (Figure 2) distorted the smooth variation of the subsurface resistivity distribution as it revealed a sharp distinction of the various resistivity layers in the subsurface. Thus the smoothing of model resistivity inversion option is suitable for areas with smooth variation of the subsurface resistivity as it produces a model with smooth variation of resistivity thus confirming the claim of previous researchers (Loke and Barker, 1996; Loke, 2001).

Smooth variation of resistivity is as a result of absence of interface like soil-bed rock interface due to absence of anomalies. For subsurface bodies with sharp boundaries such as the soil-bedrock interface or massive homogenous bodies, the default smoothness-constrained inversion option as well as the smoothing of model resistivity inversion option tends to smear the boundaries thus giving rise to a blurred view of the boundaries. The blocky or robust inversion method produces a model with sharp interfaces between different regions with different resistivity values and is thus more suitable for areas where such a geological situation exists. As could be seen in Figure 2, the blocky inversion method produced a model that clearly and better distinguished the various resistivity layers compared to the other inversion methods. Obviously, the blocky inversion methods gave a clearer view and distinct boundary of the anomaly represented with the purple colour resistivity value in smoothness constrained inversion.

5. CONCLUSION

2-D Electrical resistivity survey was successfully carried out in the study area which revealed the uniqueness of both methods of interpretation in the study area. The data sets were successfully inverted into a 2-D image of the subsurface using the smooth and blocky inversion methods from which it is clearly shown that, the smoothing of model resistivity inversion method is suitable for areas with smooth variation of subsurface resistivity while the blocky inversion method is suitable for areas with sharp boundaries between different regions with different resistivity values.

6. ACKNOWLEDGMENT

The authors wish to acknowledge the assistance and contributions of the technical staff during the field work toward the success of this work. The authors are also grateful to all the anonymous reviewers and editors whose comments improved the quality of this manuscript.

7. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

REFERENCES

Amidu, S.A. and Dunbar, J.A. (2008). An evaluation of the electrical-resistivity method for water-reservoir salinity studies. *Geophysics*, 73, 4, pp. G39 – G49.

Cardarelli, E. (2002). Geoelectric survey by ERT to investigate marble ruins at the Roman forum, Rome, Italy. *European Journal of Environmental and Engineering Geophysics*, 7, pp. 219 – 228.

Claerbout, J.F. and Muir, F. (1973). Robust modeling with erratic data. *Geophysics*, 38, pp. 826 - 844.

Constable, S.C., Parker, R.L. and Constable, C.G. (1987). Occam's inversion: A practical algorithm for generating smooth models from electromagnetic sounding data. *Geophysics*, 52, pp. 289 – 300.

Dahlin, T. (1996). 2D resistivity surveying for environmental and engineering applications. *First Break*, 14, pp. 275 – 284.

Dahlin, L.T., Forsberg, K., Nilsson, A. and Flyhammer, P. (2006). Resistivity Imaging for Mapping of Groundwater Contamination at the Municipal Landfill La Chureca, Managua, Nicaragua. Near Surface 2006, 12th European Meeting of Environmental and Engineering Geophysics, 4–6 September, Helsinki, Finland.

Daily, W. and Ramirez, A.L. (2000). Electrical imaging of engineered hydraulic barriers. *Geophysics*, 65(1), pp. 83 – 94.

Dey, A. and Morrison H.F. (1979). Resistivity modeling for arbitrary shaped two-dimensional structures. *Geophysical Prospecting* 27, pp. 1020 – 1036.

Ellis, R.G. and Oldenburg, D.W. (1994). Applied geophysical inversion. *Geophysical Journal International*, 116, pp. 5 – 11.

Georgaki, I., Soupios, P., Sakkas, N., Ververidisa, F., Trantasa, E., Vallianatos, F. and Manios T., (2008): Evaluating the use of electrical resistivity imaging technique for improving CH_4 and CO_2 emission rate estimations in landfills. *Science of the Total Environment*, 389, pp. 522 – 531.

Kemma, A., Vanderborght, J., Kulessa, B. and Vereecken, H. (2002). Imaging and characterization of subsurface solute transport using electrical resistivity tomography (ERT) and equivalent transport model. *Journal of Hydrology*, 267, pp. 125 – 146.

Lines, L.R. and Treitel, S. (1984). Tutorial: A review of least-squares inversion and its application to geophysical problems. *Geophysical Prospecting*, 32, pp. 159 – 186.

Loke, M.H and Barker, R.D. (1996): Practical Techniques for 3D resistivity Surveys and data Inversion. Geophysical Prospecting; 44, pp. 499-524.

Loke, M.H. (2001). Electrical imaging survey for environmental and engineering studies: A practical guide to 2D and 3D surveys. www.geoelectrical.com p. 62.

Loke, M.H. (2004). Tutorials: 2D and 3D electrical imaging surveys, Geotomo software, viewed 6 May-2015. www.geoelectrical.com.

Loke, M.H. (2010). RES2DINV ver. 3.59 for Windows XP/Vista/7, Rapid 2-D Resistivity & IP Inversion using the least-squares method. Geotomo Software. Manual.

Schrott L. and Sass .O. (2008): Application of field geophysics in geomorphology: Advances and limitations exemplified by case studies. *Geomorphology*, 93, pp. 55 – 73.

Sechman, H., Mościcki, W.J. and Dzieniewicz, M. (2013). Pollution of near-surface zone in the vicinity of gas wells. *Geoderma*, 197–198, pp. 193 – 204.

Shima, H. (1992). 2-D and 3-D resistivity image reconstruction using crosshole data. *Geophysics*, 57, pp. 1270 – 1281.

Shima, H. (1995). Inversion in electrical and electromagnetic exploration. *Journal of Geography*, 104, pp. 952 – 971.

Silvester, P.P. and R.L. Ferrari, (1990). *Finite elements for electrical engineers* (2nd. ed). Cambridge University Press, Cambridge, pp. 770

Suzuki, K and Ohnishi, H. (1995). Application of the electrical method to field survey. Annual Inter-national Meeting, SEG, Expanded Abstracts, pp. 338 – 343.

Wolke, R. and Schwetlick, H. (1988). Iteratively reweighted least squares algorithms, convergence analysis, and numerical comparisons: *SIAM Journal of Scientific and Statistical Computations*, 9, pp. 907 – 921.

Zhou, Q.Y. (2007). A sensitivity analysis of DC resistivity prospecting on finite, homogeneous block and columns. *Geophysics*, 72, pp. F237 – F247.