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Reliability Analysis of Slender Reinforced Concrete Column under Biaxial Loading

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ABSTRACT

Limit state equations for slender reinforced concrete column subjected to biaxial bending, considering axial, deflection and bending mode of failure were derived. Their reliability levels were then evaluated as the axial mode of failure is 4.52 (P_f = 1.08E-03), deflection mode of failure is 3.47 (P_f = 1.38E-03) and biaxial bending mode of failure is 4.61 (P_f = 1.84E-03). All basic design variables involved were treated as random variables with their statistical distribution adopted from literature. The results showed that safety index decreases with increase in load and largely depends on their variable parameters. Sensitivity analysis was also carried out on the column section to determine how the design variables behave in respect of their safety indices. The result showed that safety index increased with increase in variables such as characteristic strength of concrete, area of concrete section, characteristic strength of steel reinforcement, area of steel reinforcement, width of concrete section and overall depth of concrete section while it decreased with increase in the applied load. The design becomes critical for all the variables at the applied axial load of 2243 kN and beyond. This is in line with the safety indices range of 3.70 ($Pf \approx 10^{-6}$) to 4.70 ($Pf \approx 10^{-6}$) recommended by Joint Committee on Structural Safety for structure with large consequences of failure.

1. INTRODUCTION

Columns are vertical structural elements with height much greater than their breadth that majorly transmit compressive loads and moments from beams, slabs, other columns and wind loads to the substructure. Current design provisions for columns are based on deterministic approach largely in accordance with national or international standards such as BS8110 (1997). Due to inherent uncertainties in materials and loadings, the codes/standards provide for safety factors to account for them. This however, could lead to a conservative design as observed by Abdulwahab and Uche (2016). Hence the most rational approach to analysis of safety is by using the stochastic models where the effects of the uncertainties can be accounted for and treated in a way that would result in having structures with high level of reliability.

Variations and uncertainties in design processes and operation conditions causes variation in its performance and durability. Many sources of uncertainty are inherent in structural design. Despite what is often thought, the parameters of the loading and the load-carrying capacities of structural members are not deterministic quantities (that is, quantities which are perfectly known). They are random variables, and thus absolute safety (or zero probability of failure) cannot be achieved. Consequently, structures must be designed to serve their function with a finite probability of failure (Melchers, 1999).

A structure is only as strong as its ‘weakest element’, hence members with good measure of reliability do contribute significantly to overall structural reliability (Abubakar and Ma’aruf, 2014). The reliability of a structure is its ability to fulfil its design purpose for some specified design lifetime. It is often understood to equal the probability that a structure will not fail to perform its intended function. The term “failure” does not necessarily mean catastrophic failure but is used to indicate that the structure does not perform as desired as observed by Uche and Afolayan, (2008). They described reliability as the probability or likelihood of a structure performing its purpose adequately for a period of time intended under the operating conditions encountered. The problem associated with the traditional method of ensuring safety can be resolved by rendering broad, general concepts, such as uncertainties and risks, into precise mathematical terms that can be operated upon consistently. This approach essentially forms the basis of reliability-based design. Uncertain engineering quantities (e.g. loads, capacities) are modelled by random variables, while design risk is quantified by the probability of failure.

A structural element will be considered to have failed if its resistance (R) is less than the stress resultant (S) acting on it (Nowak and Collins, 2000). Once the uncertainties in the resistance and the stress resultant have been modelled as random variables, the probability of failure (Pf) can be evaluated as follows (Melchers, 1999):

$$P_f = Prob(R < S) \tag{1}$$

$$= \int_{-\infty}^{+\infty} F_R(x) f_s(x) dx \tag{2}$$

Where $f_s(x)$ = Probability density function of S and $F_R(x)$ = Probability distribution function of R.

According to earlier design standard such as BS8110, (1997), it states that the limit state design procedure is a semi-probabilistic design process that uses partial safety factors (load and resistance factors) determined empirically, to ensure that an acceptably low probability of failures is achieved for the design cases to which the partial safety factors apply. It was also proved that the safety factors are empirical quantities derived from a number of analytical processes in which uncertainties may result. The accumulation of these uncertainties could make the design unsafe and uneconomical.

Therefore, in probabilistic design any uncertainties about a variable should be given due consideration. This has made it necessary in the field of structural engineering to review the traditional way of reinforced concrete column design by performing stochastic analysis and also determining the resistant load and safety index. These will help to ascertain the uncertainties which are likely to occur during the life of the structure and the general mode of failure for the structural reinforced concrete column under biaxial loading.

2. MATERIALS AND METHODS

2.1. Materials

FORM5 computer package which uses an algorithm linked to FORTRAN was used in this study to compute the implied safety levels for the different limit state functions, outlining the design criteria for reliability study of slender reinforced concrete column in accordance with BS 8110 (1997). It handles up to 60 uncertain variables (x-variables) and can perform up to 40 iterations to achieve convergence.

2.2. Methods

A typical slender reinforced concrete column of a three (3) storey commercial building with the following parameters was considered in which the deterministic and stochastic design were carried out in accordance with (BS 8110, 1997):

Size of column = 400 x 300 mm
Heights of column = 6.5 m
Direct load on column N = 1500 kN

2.2.1. Derivation of safe design parameters

In order to resist the loads, the resistance properties must be carefully chosen. This selection may not only revolve about the derivation of the ultimate strength equation of slender column section but also aid in deriving the limit state expression of the various failure modes considered in its loading. Hence the limits state equation considered for this design are axial failure, deflection failure and biaxial bending failure modes and are as given in Equations 3 to 6.

Axial failure mode:

$$g(x) = g\,(N, A\text{sc}, A\text{c}, f\text{y}, f\text{cu}) = 0.45f\text{cu}\,A\text{c} + 0.87f\text{y}A\text{sc} - N \tag{3}$$

Where:
$f$cu is the characteristic strength of concrete
Ac is the cross-sectional area of concrete
Asc is the cross-sectional area of steel reinforcement
fy is the characteristic yield strength of steel
N is the applied axial load

Deflection failure mode:

$$g(x) = g(\beta, \text{K}, \text{h}) = \frac{l_{ex}}{2000} - \beta Kh \qquad \text{(along y-axis)} \tag{4}$$

$$g(x) = g(\beta, \text{K}, \text{b}) = \frac{l_{ey}}{2000} - \beta Kb \qquad \text{(along y-axis)} \tag{5}$$

Where:
β is the coefficient to determine modified bending moments in biaxial bending
K is the factor governing deflection of column due to slenderness
l_{ex} is the effective height for consideration of slenderness about x-axis
l_{ey} is the effective height for consideration of slenderness about y-axis
b is the width of rectangular column section – dimension perpendicular to y-axis

h is the overall depth of rectangular column section – dimension perpendicular to x-axis
Biaxial Bending Failure Mode:

$$G(x) = g\,(N, A\text{sc}, A\text{c}, f\text{y}, f\text{cu}, M\text{c}) = \left(\frac{Mc}{M\text{uz}} + \frac{N}{0.45\text{fcuAc}+0.87\text{fyAsc}}\right) \leq 1 \quad (6)$$

Mc is the applied moment

2.2.2. FORTRAN subroutine

The FORTRAN subroutine was written to accommodate different modes of failure of reinforced concrete columns subjected to biaxial loading in FORTRAN language and linked with FORM5 to solve the limit state in axial load, deflection and biaxial moment. The flow chart for the subroutine is shown in Figure 1. The FORM 5 was then launched via the command prompt of the computer.

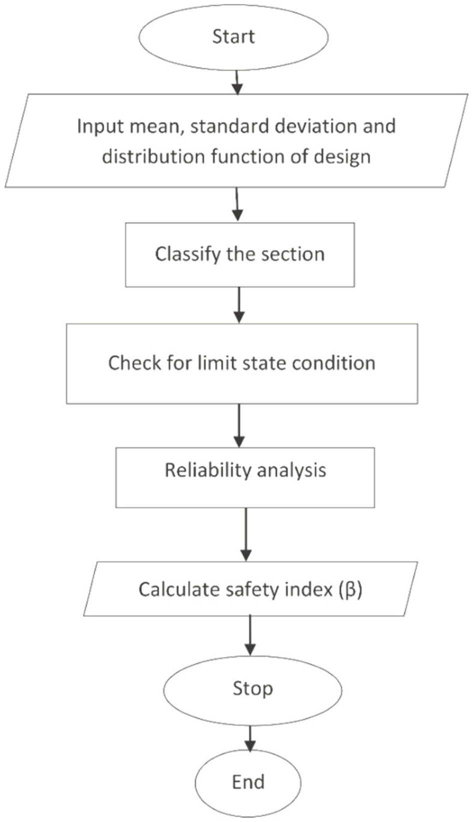

Figure 1: Chart for reliability analysis using FORM 5

3. RESULTS AND DISCUSSION

3.1. Reliability Analysis

Using the data generated from the reliability evaluation of the various limit state conditions (subroutines), the safety indices for axial mode of failure, deflection mode of failure and biaxial bending mode of failure were recorded for the slender reinforced concrete column under biaxial bending as presented in Figure 2. The reliability index obtained were in line with the safety indices range of 3.70 to 4.70 recommended by

JCSS (2000) for a structure with large consequences of failure for axial and biaxial bending mode of failure whereas for deflection mode of failure it's conservative when considering the target safety indices range of 1.30 to 2.30 recommended by JCSS (2000) for structure with serviceability limit state.

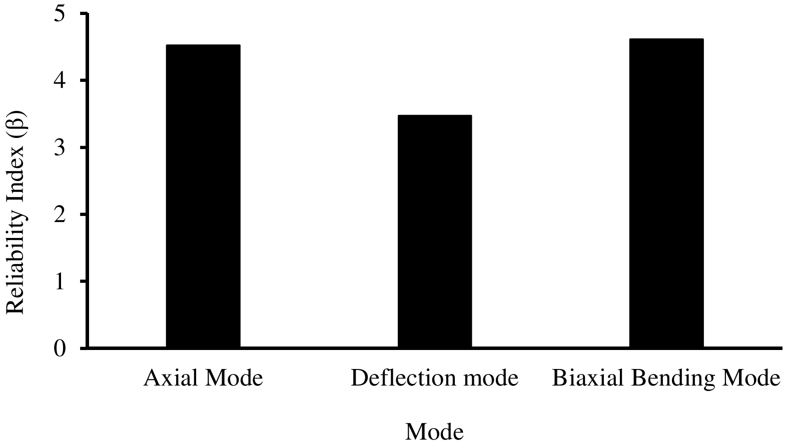

Figure 2: Safety indices of slender reinforced concrete column

3.2. Sensitivity Analysis

The safety indices for the sensitivity analysis for axial mode of failure, deflection mode of failure and biaxial bending mode of failure were recorded for the slender reinforced concrete column under biaxial bending as presented in Figure 3 to Figure 8. From the results presented in Figure 3, it was observed that the safety indices increased with increase in characteristics compressive strength at a given load which was as a result of increase in the concrete strength as reported by Abdullahi *et al.* (2019). It was also observed that the characteristics compressive strength of 20 N/mm^2 to 35 N/mm^2 at axial load 1500 kN with respective safety indices of 4.08 to 4.70 would preferably give a safe design.

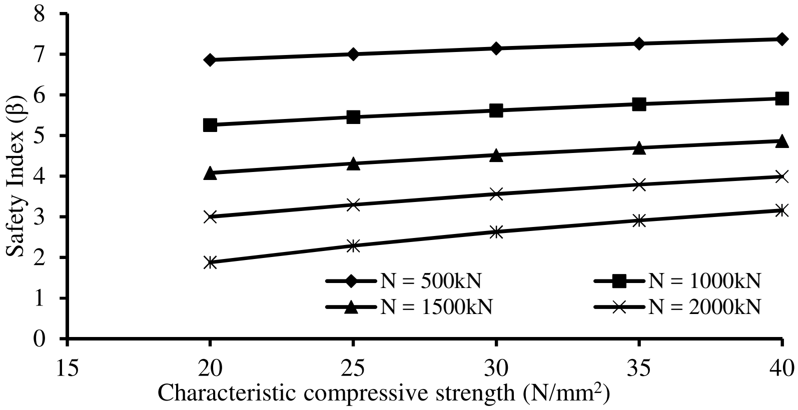

Figure 3: Safety indices for varying characteristics compressive strength at various applied axial load for axial mode of failure

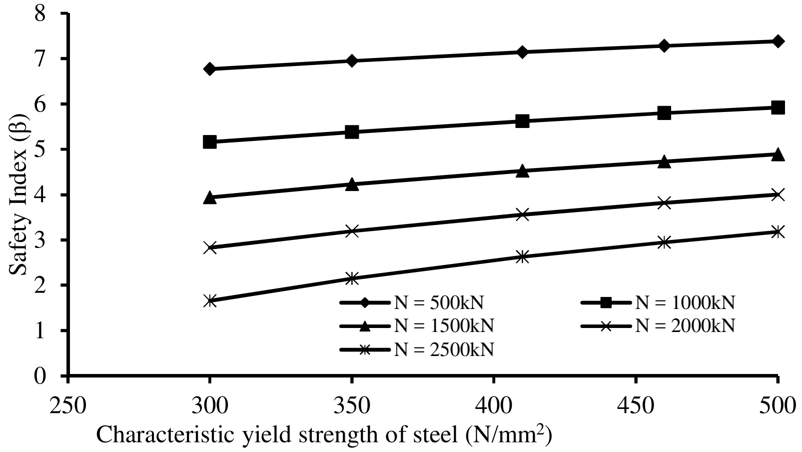

Figure 4: Safety indices for varying characteristics yield strength at various applied axial load for axial mode of failure

It is obvious that the design is critical at all axial load when range 25 N/mm^2 to 30 N/mm^2 was used as the characteristic strength at axial load of 2000 kN with corresponding safety indices of 3.30 and 3.56, whereas it became conservative when a range of 20 N/mm^2 to 40 N/mm^2 was used. This is in consonance with the safety indices range of 3.70 to 4.70 recommended by JCSS (2000) for a structure with large consequences of failure.

It was observed from Figure 4, that the safety indices increased with increase in characteristics strength of steel reinforcement in the sensitivity analysis. For the design to be safe, preferably characteristic strength of steel range of 300 N/mm^2 to 410 N/mm^2 at axial load of 1500 kN should be adopted with corresponding safety indices of 3.94 to 4.52.

When 350 N/mm^2 and 410 N/mm^2 are used as the characteristic strength of steel reinforcement at an axial load of 2000 kN, the design becomes critical while it becomes conservative when 300 N/mm^2 to 460 N/mm^2 at an axial load of 1000 kN. This is also in line with the safety indices range of 3.70 to 4.70 recommended by JCSS (2000) for a structure with large consequences of failure.

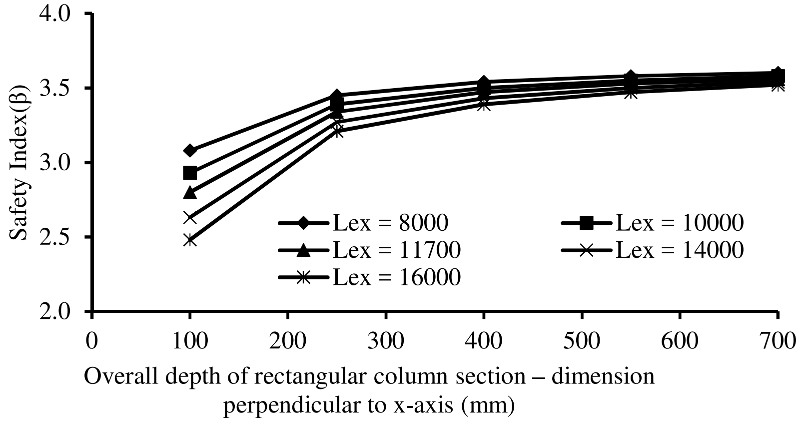

Figure 5: Safety indices for varying overall depth of rectangular column at various effective height for consideration for slenderness about x-axis for deflection mode of failure

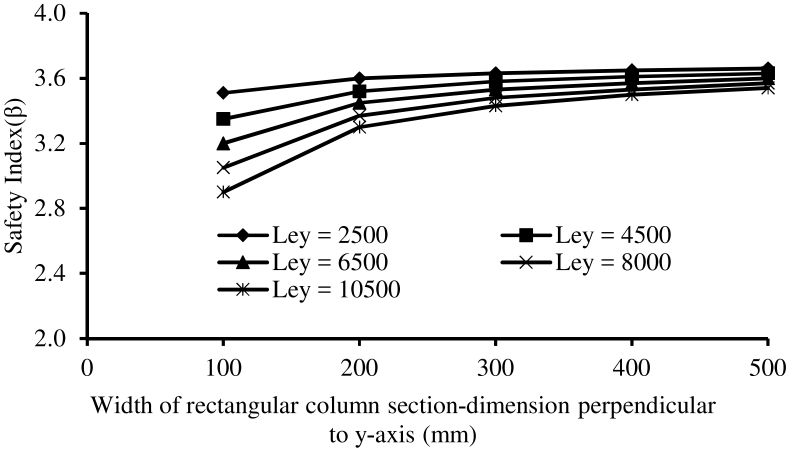

Figure 6: Safety indices for varying width of rectangular column at various effective height for consideration for slenderness about y-axis for deflection mode of failure

It was recorded from Figure 6 the safety indices decreased with increase in effective length (lex) along x-axes and safety indices also increased with increase in overall depth of rectangular column section at a giving load. The design is conservative for all the variation of effective length with overall depth of rectangular column section with corresponding safety indices ranging from 2.48 to 3.60. This is in line with the target safety indices range of 1.30 to 2.30 recommended by JCSS (2000) for structure with serviceability limit state. It is also obvious that the design is conservative of all variation for effective length and width of rectangular column section with corresponding safety indices ranging from 2.90 to 3.66. This is in consonance with the safety indices range of 1.30 to 2.30 recommended by JCSS (2000) for a structure with serviceability limit state.

From the result presented in Figure 7, it was observed that the safety indices increased with increase in characteristics compressive strength at a given load which was as a result of increase in the concrete strength. It was also observed that the characteristics compressive strength of 20 N/mm^2 to 35 N/mm^2 at axial load 1500 kN with respective safety indices of 4.16 to 4.79 would preferable give safe design.

It was also recorded that the design is critical at all axial load when range 25 N/mm^2 to 30 N/mm^2 was used as the characteristic strength at axial load of 2000 kN with corresponding safety indices of 3.37 and 4.07, whereas it become conservative when range 20 N/mm^2 to 40 N/mm^2 was used. This is in consonance with the safety indices range of 3.70 to 4.70 recommended by JCSS (2000) for a structure with large consequences of failure.

Figure 8 demonstrated that the safety indices rise with increase in characteristics strength of steel reinforcement. Hence, for the design to be safe, preferably characteristic strength of steel range of 300 N/mm^2 to 410 N/mm^2 at axial load of 1500 kN should be adopted with corresponding safety indices of 4.02 to 4.61. When 350 N/mm^2 and 410 N/mm^2 are used as the characteristic strength of steel reinforcement at an axial load of 2000 kN the design becomes critical, while it becomes conservative when 300 N/mm^2 to 460 N/mm^2 at an axial load of 1000 kN. This is in agreement with the safety indices range of 3.70 to 4.70 recommended by JCSS (2000) for a structure with large consequences of failure.

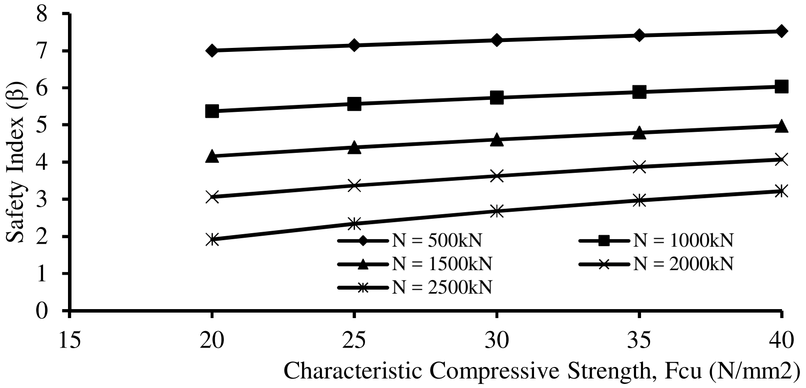

Figure 7: Safety indices for varying characteristic compressive strength for various applied axial load for biaxial bending mode of failure

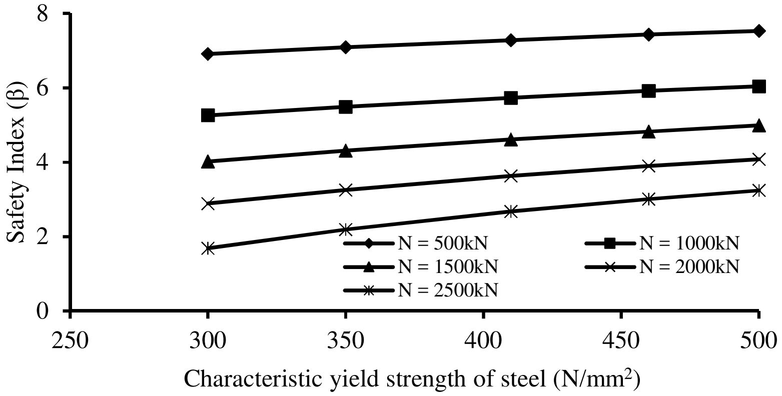

Figure 8: Safety indices for varying characteristic yield strength of steel reinforcement for various applied axial load for biaxial bending mode of failure

4. CONCLUSION

The following conclusions were drawn after the reliability assessment on the existing deterministic design of reinforced concrete column under biaxial bending in accordance with BS8110 (1997).

i) Limit state equations for axial, deflection and biaxial bending mode of failure for slender reinforced concrete column under biaxial bending were established.
ii) The safety index for axial mode of failure is 4.52 (P_f=0.18E-02), deflection mode of failure is 3.47 (P_f =0.138E-02) and biaxial bending mode of failure is 4.61 (P_f=0.184E-02) for the slender reinforced concrete column under biaxial bending.
iii) The sensitivity analysis shows how the design variables behave in respect of their safety indices. For this analysis, safety index increased with increase in variables such as characteristic strength of concrete, area of concrete section, characteristic strength of steel reinforcement, area of steel reinforcement, width of concrete section and overall depth of concrete section while it decreases with

increase in applied load. The design becomes critical for all the variables at the applied axial load of 2243 kN and beyond.

iv) The design results are consistent when compared with the safety indices recommended by the Joint Committee of Structural Safety Code (JCSS, 2000) for large consequences of failure

5. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

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