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Analysis of Fluid Flow and Heat Transfer Over a Non-isothermal Inclined Flat Plate using the Modified Merk Series of Chao and Fagbenle

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ABSTRACT

The Merk-Chao-Fagbenle (MCF) procedure is employed in the calculation of fluid flow and heat transfer over a non-isothermal inclined flat plate. The series developed with this procedure and the result of the wall derivatives of temperature was used directly for the heat transfer calculations. At Prandtl number (Pr) = 0.7 and pressure gradient (Λ) =0, the wall derivatives of temperature functions were taken from the table provided. This table was the result derived from the calculations of temperature wall derivatives by the above procedure, are input into the modified Merk's series of Chao and Fagbenle. The heat transfer calculations were carried out and the results compared with the earlier results of Chao and Fagbenle at a temperature exponent (a) = 0. The results showed that the local Nusselt number (Nu) for Pr=0.7, Λ=-0.15, -0.10, 0.05, 0.10 for Fagbenle were 0.9921, 1.2319, 1.8968, 1.3695 respectively and for the current work, 1.041214, 1.2361, 1.8927, 1.3641 respectively showing that the results are in close agreement. Other results at various Λ's and a's are presented in tables and graphs. The fact that these result are in close agreement confirms the accuracy of the MCF procedure.

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1. INTRODUCTION

Numerous industrial and manufacturing process rely on the thermal boundary layer concept such as hot rolling, wire drawing, fiber-glass and paper production, gluing of labels on hot bodies, the design of heat exchangers, extrusion of plastic sheets, calculations of heat transfer over turbine blades, etc. When a cooler fluid flows around a hot body, the temperature of the fluid will rise in a thin layer around the body and in the wake behind it. This thin layer is known as the thermal boundary layer. In this layer, flow and thermal phenomena interact non-linearly and are governed by the so-called thermal boundary layer equations.

The series-based expansion methods which were first employed by Blasius, (1908), to the boundary layer fluid flows has continued to be developed rapidly and employed by such researchers as Gortler, (1957) and Merk, (1959) and this has shown that there is serious need for the method in the future. Generally speaking, the Blasius series is quite effective for fluid flows over blunt objects like cylinders. In the case of slender bodies, an excessive number of terms would be required in the polynomial representation and the series surfers from slow convergence. A procedure which belongs to the category of 'wedge methods' and which provides a rigorous refinement of the local similarity concept is that of Merk. An essential feature of the Merk scheme is that it makes possible rapid calculations of the significant boundary layer quantities (skin friction, heat transfer, mass transfer, etc) with the aid of a limited number of universal functions which can be tabulated once and for all (Chao and Fagbenle, 1974). An advance in the accuracy of boundary layer series solution was therefore made possible by Merk, who refined the 'wedge method' proposed by Meksyn, (1961) by choosing to treat the wedge parameter (Λ) as an independent variable rather than as a function of the stream wise coordinate, (ξ). Thus the Merk series were expanded about the local similarities solution rather than the forward stagnation point of the body as it has been the convention in the past. However, an error in the form of the series presented by Merk was found by independent researchers, (Fagbenle, 1973; Chao and Fagbenle 1974). Chao and Fagbenle (1974) put forth a corrected form of Merk series and used it to perform a universal, laminar boundary layer analysis for the forced flow of Newtonian fluids over isothermal bodies. Since then, the MCF approach has been used with success for a family of boundary layer solutions

Researchers, including Cameron et al. (1991), used the MCF equation to investigate mixed, forced and natural convection from two-dimensional or axisymmetric bodies of arbitrary contour. Meissner et al. (1994), used the same method for mixed convection to power-law fluids from two-dimensional or axisymmetric bodies, with huge success. Recently Amoo et al. (2020) presented a comparative analysis of numerical methods including the MCF series applied to non-similar boundary layer-derived infinite series equation. By introducing the two-parameter MCF series into the transformed boundary layer equations for non-isothermal surfaces, there resulted a set of ordinary differential equations with three parameters, the pressure gradient (Λ), the Prandtl number (Pr) and the temperature parameter (a). Therefore, by assigning numerical values to these parameters, this set of equations was solved so that the results for the flow field and the heat transfer was expressed in terms of universal functions, (Falana, 2013; Falana and Fabgenle, 2014). Solutions were obtained for combinations of the non-isothermal parameters for various combinations of the temperature exponent, the Prandtl number, and the pressure gradient. Specific flow situations may be rapidly analyzed using these tabulated solutions.

In this paper, the universal wall derivative of temperature functions derived from the MCF series of Falana, (2013) was applied to the analysis of fluid flow and heat transfer over a non-isothermal inclined flat plate.

2. METHODOLOGY

2.1. Problem Formulation

The Merk-Chao-Fagbenle method is strictly applicable to incompressible, uniform property, laminar boundary layer flows. However, the MCF equations governing the flow and heat transfer for a non-isothermal surfaces are not re-derived here. They are solved for wall derivatives of universal temperature functions (θ'_i) using FORTRAN 77 and are tabulated once and for all, (Falana, 2013). For pressure gradient, (Λ=0), and Prandtl, (Pr=0.7), the table for these values is made available in Table 1.
The MCF series employed in this work is stated as follows:

$$\theta(\xi,\eta) = \theta_0((\wedge,\eta) + 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_1(\wedge,\eta) + \gamma\theta_2(\wedge,\eta) + \gamma 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_3(\wedge,\eta) + 4\xi^2\left(\frac{d^2\wedge}{d\xi^2}\right)\theta_1(\wedge,\eta) + \cdots \tag{1}$$

The calculation begins with an evaluation of the dimensionless stream wise coordinate (ξ) and the wedge variable or the pressure gradient ($\wedge$) according to Equations (2) and (3).

$$\xi = \int_0^x \frac{U(x)}{U_\infty}\left(\frac{r}{L}\right)^2 \frac{dx}{L} \tag{2}$$

$$\wedge = \frac{2\xi}{U}\frac{du}{d\xi} = 2\left(\frac{L}{r}\frac{U_\infty}{U(x)}\right)^2\left\{\int_0^x \left(\frac{r}{L}\right)^2 \frac{U(x)}{U_\infty}\frac{dx}{L}\right\}\frac{d\left(\frac{U}{U_\infty}\right)}{(x/L)} \tag{3}$$

The reference velocity (U_∞) and length (L) may be any convenient quantities appropriate to the problem under consideration; they are constants. Whenever feasible, it is desirable to express $\frac{U(x)}{U_\infty}$ and $\frac{r}{L}$ as a polynomial of x/L.

Table 1: Wall derivatives of temperature functions for Pr = 0.7

$\wedge$	$\theta_0'(\wedge,0)$	$\theta_1'(\wedge,0)$	$\theta_2'(\wedge,0)$	$\theta_3'(\wedge,0)$
-0.15	0.36437340	-0.73064340E-10	-0.47564870E-05	-0.28152810E-14
-0.10	0.38697190	-0.11286630E-09	-0.70105800E-05	-0.10904380E-13
-0.05	0.40223690	-0.11731800E-09	-0.87854970E-05	-0.13668280E-13
0.00	0.41391300	-0.12072300E-09	-0.10302160E-04	-0.16030270E-13
0.05	0.42340000	-0.12348970E-09	-0.11648920E-04	-0.18127830E-13
0.10	0.43139610	-0.12582120E-09	-0.12872300E-04	-0.20033400E-13
0.20	0.44438450	-0.12960830E-09	-0.15052850E-04	-0.23430270E-13
0.30	0.45469960	-0.13261570E-09	-0.16978330E-04	-0.26430190E-13
0.40	0.46323390	-0.13510350E-09	-0.18719960E-04	-0.29144000E-13
0.50	0.47049320	-0.13721970E-09	-020321000E-04	-0.31639060E-13
0.60	0.47679620	-0.13905670E-09	-0.21810020E-04	-0.33959780E-13
0.70	0.48235520	-0.14067680E-09	-0.23207220E-04	-0.36137600E-13
0.80	0.48731890	-0.14212320E-09	-0.24527370E-04	-0.38195540E-13
0.85	0.48961240	-0.14279140E-09	-0.25162150E-04	-0.39185140E-13
0.90	0.49179500	-0.14342750E-09	-0.25781650E-04	-0.40150950E-13
0.95	0.49387680	-0.14403400E-09	-0.26386930E-04	-0.41094650E-13
1.00	0.49586610	-0.14461340E-09	-0.26978860E-04	-0.42017570E-13

For a two-dimensional boundary layers, one sets $r = L$. Hence, the expression r/L drops out of the computation, and as a result of this, straightforward algebraic equations for ξ and $\wedge$ can be obtained and the determination of $2\xi\frac{d\wedge}{d\xi}$ and $4\xi^2\frac{d\wedge^2}{d\xi^2}$ and the expression for the temperature parameter, (γ) in Equation (1) becomes simple. At the forward stagnation point, $x = \xi = 0$ and $\wedge = 1$ or $\frac{1}{2}$, corresponding respectively to the two-dimensional or axisymmetrical boundary layers. The local heat transfer coefficient is given by:

$$Nu = \frac{hL}{k} = \frac{q_w L}{k(T_w - T_\infty)} = \frac{-L\frac{dT}{dy} \text{ @ } y = 0}{T_\infty - T_w}$$

$$Nu/_{Re^{\frac{1}{2}}} = \frac{r}{L}\frac{U(x)}{U_\infty}(2\xi)^{-\frac{1}{2}}[-\theta'(\Lambda,0)] \tag{4}$$

With the aid of Table 1, the local heat transfer coefficient can be obtained by substituting $\eta = 0$ at the wall in Equation (3) as follows:

A. Falana / Nigerian Research Journal of Engineering and Environmental Sciences
7(1) 2022 pp. 384-392

$$NuRe^{-\frac{1}{2}} = \frac{r}{L}\frac{U(x)}{U_\infty}(2\xi)^{-\frac{1}{2}}\Big[\theta_0'(\wedge,0) + 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_0'(\wedge,0) + \gamma\theta_2'(\wedge,0) + \\ +\gamma 2\xi\left(\frac{d\wedge}{d\xi}\right)\theta_0'(\wedge,0) + 4\xi^2\left(\frac{d^2\wedge}{d\xi^2}\right)\theta_0'(\wedge,0) + \cdots\Big] \quad (5)$$

The universal functions (θ'_i) which are made available in Table1 depend on the Prandtl number as a parameter. Making use of the MCF procedure, the local heat transfer for flow and heat transfer over an inclined non-isothermal plate containing three parameters is then calculated.

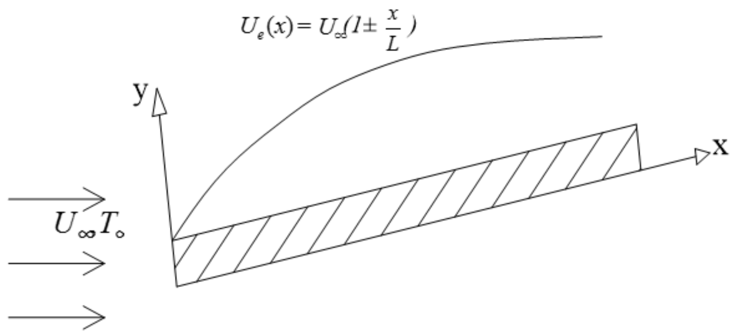

Figure 1: Fluid Flow along an inclined flat plate

A well-known boundary layer flow is that over a flat surface inclined at a small angle to the mainstream. The velocity at the edge of the boundary layer varies linearly with the distance along the surface according to Equation (6) (Chao and Fagbenle, 1974).
In Figure 1, L is the total length of the inclined plate and x is any point on the plate, therefore the velocity of fluid within the boundary layer is given by:

$$U(X) = U_\infty(1 \pm X) \quad (6)$$

where $X = x/L$ and $\frac{1}{L}\frac{d}{dx}\left[\frac{U(x)}{U_\infty}\right] is$ a constant
In Equation (6) and others which follow, the positive sign is for accelerated flows and the negative sign is for retarded flows.
If $M = 1 \pm X$, the ratio of the fluid velocity within the boundary (U(x)) and the velocity at the edge of the boundary layer, (U_∞) is:

$$\frac{U(x)}{U_\infty} = M \quad (7)$$

From Equation (2):

$$2\xi = \pm(M^2 - 1) \quad (8)$$

From Equation (3):

$$\wedge = 1 - M^{-2} \quad (9)$$

Therefore in Equation (1):

$$2\xi\frac{d\wedge}{d\xi} = 2M^{-2}(1 - M^{-2}) \quad (10)$$

and

$$4\xi^2\frac{d^2\wedge}{d\xi^2} = 8M^{-2}(1 - M^{-2})^2 \quad (11)$$

The local heat transfer coefficient over a surface of non-uniform temperature is given by Equation (5), which upon introducing the expressions for $\xi, 2\xi\frac{d\wedge}{d\xi}, 4\xi^2\frac{d^2\wedge}{d\xi^2}$, etc, becomes:

$$NuRe^{-1/2} = \frac{M}{\sqrt{\pm(M^2-1)}}[\theta_0'(\wedge,0) + 2M^{-2}(1-M^{-2})\theta_1'(\wedge,0) + \gamma\theta_2'(\wedge,0) + 2\gamma M^{-2}(1-M^{-2})\theta_3'(\wedge,0) - 8M^{-2}(1-M^{-2})^2\theta_1'(\wedge,0) + \cdots] \quad (12)$$

where $Re = U_\infty L/\nu$, the Re= the Reynolds number and ν= kinematic viscosity
The temperature parameter for the non-uniform surface for the inclined flat plate is defined as:

$$\gamma = \frac{2\xi\frac{d}{d\xi}(T_w(x)-T_\infty)}{(T_0-T_\infty)} \quad (13)$$

Stating $T_w(x)-T\infty = (T_0-T\infty)e^{\frac{ax}{L}}$, initial temperature variation on the surface of the flat plate, Falana, 2013. From Falana, (2013), and using Equations (6), (7) and (8):

$$\gamma = \frac{[(1+{}^x/_L)^2-1]\frac{d}{d\xi}(e^{ax/L})}{e^{ax/L}} \quad (14)$$

The temperature parameter is given by:

$$\gamma = \frac{[(1+{}^x/_L)^2-1]\frac{d}{dx}(e^{ax/L})\frac{dx}{d\xi}}{e^{ax/L}} \quad (15)$$

which reduces to:

$$\gamma = \frac{a}{(1+{}^x/_L)}\left[(1+{}^x/_L)^2-1\right] \quad (16)$$

Using Equation (6):

$$\gamma = a(M-M^{-1}) \quad (17)$$

where $M = (1+{}^x/_L)$.
Putting the value of the temperature parameter, (Equation 17) into Equation (5), the Nusselt number becomes:

$$NuRe^{-1/2} = \frac{M}{\sqrt{(M^2-1)}}[\theta_0'(\wedge,0) + 2M^{-2}(1-M^{-2})\theta_1'(\wedge,0) + a(M-M^{-1})\theta_2'(\wedge,0) + 2a(M-M^{-1})M^{-2}(1-M^{-2})\theta_3'(\wedge,0) - 8M^{-2}(1-M^2)^2\theta_1'(\wedge,0) + \cdots] \quad (18)$$

3. RESULTS AND DISCUSSION

The results were generated from Equation (18). The values of the universal wall derivatives of temperature functions (θ'_i) were taken from Table 1. The equation was programmed using FORTRAN 77, the results were imported into an EXCEL Worksheet for graphing. In order to study the influence of all the parameters (the Prandtl number (Pr), the temperature exponent (a) and the pressure gradient, (Λ) on the thermal fields, a selected set of graphical and tabulated results are presented. It is noticed here that an increase in the parameter a, ($a > 0$) indicates a decrease in the fluid viscosity. Extensive calculations have been performed to obtain the local Nusselt number for a wide range of the parameters which are tabulated (Falana, 2013). For the value of $a = 0$, which corresponds to an isothermal wall condition, the results have been compared with the one reported by Chao and Fagbenle (1974) as shown in Table 2, and it is found to be in good agreement. This serves as a benchmark for the accuracy of our numerical procedure. It is interesting to note that the surface heat transfer rate increased with an increase in Prandtl number as shown in Tables 3 and 4. The physical reason for this trend is that at higher Prandtl numbers, the fluid thermal boundary layer becomes thinner leading to an increase in the temperature gradient (Ali, 1995). The local heat transfer coefficient,

($Nu/Re^{1/2}$) depends on the Prandtl number and the temperature exponent, (a), as shown in Tables 2-4 and Figures 2-11.

Table 2: Computations showing comparison of the local Nusselt number ($Nu/Re^{1/2}$) for flow over an inclined flat surface with (Chao and Fagbenle 1974) for Pr $= 0.7$ and temperature exponent, $a = 0$

$\frac{x}{L}$	Λ	$Nu/Re^{1/2}$	
		Chao and Fagbenle (1974)	Present work
0.0675	-0.15	0.9921	1.0412
0.0465	-0.10	1.2319	1.2361
0.0241	-0.05	1.7989	1.7943
0.0260	0.05	1.8968	1.8927
0.0541	0.10	1.3695	1.3641
0.1952	0.30	0.8391	0.8302
0.4142	0.50	0.6738	0.6654

Comparison was also made with the published data of Ali, (1995), in Tables 3 and 4 in terms of Nusselt number for Λ=0, which corresponds to the case of flat plate at zero incidence.

Table 3: Values of $NuRe^{-1/2}$ for $a = -0.2,\ \Lambda = 0$ for a flat plate

x/L	Pr	$Nu/Re^{1/2}$	
		Ali (1995)	Present work
1.0	0.70	0.3349597	0.348045
0.98	1.0	0.4147531	0.412349
0.31	10.0	1.5145560	1.511818

Table 4: Values of $NuRe^{-1/2}$ for $a = 1.0,\ \Lambda = 0$ for a flat plate

x/L	Pr	$Nu/Re^{1/2}$	
		Ali (1995)	Present work
0.28	0.70	0.4614724	0.461801
0.41	1.0	0.5799882	0.579805
0.15	10.0	2.2958920	2.294621

From Figures 2-11 it can be seen that the local heat transfer coefficient ($NuRe^{-1/2}$) depends on the temperature exponent (a) for the various Prandtl numbers considered and exerts substantial influence on the Nusselt number. In Figures 10 and 11, negative values of the local heat transfer coefficient ($Nu/Re^{1/2}$) indicate that heat flows into the surface despite the surface temperature's continual excess over the free-stream temperature. This physical mechanism could be explained as fluid particle heated to nearly the surface temperature being convected downstream to a place at which the surface temperature is lower (Ali, 1995). Heat flows into the surface and results in negative heat transfer coefficients, which means only that θ' (0) is no longer proportional to ($T_w - T_\infty$). However, positive values of $Nu/Re^{1/2}$ show that heat is transferred from the surface to the medium which results in positive heat transfer coefficient. It is then clear that there is enhancement of $Nu/Re^{1/2}$ for increase in the value of the temperature exponent (a) which indicates that there is improvement in the convection of the heated fluid particle away from the surface downstream where the surface has a temperature lower than that of upstream. Thus, heat always flows from the surface to the medium and enhances the heat transfer coefficient; however, the opposite is true for negative temperature exponent (a<0).

A. Falana / Nigerian Research Journal of Engineering and Environmental Sciences
7(1) 2022 pp. 384-392

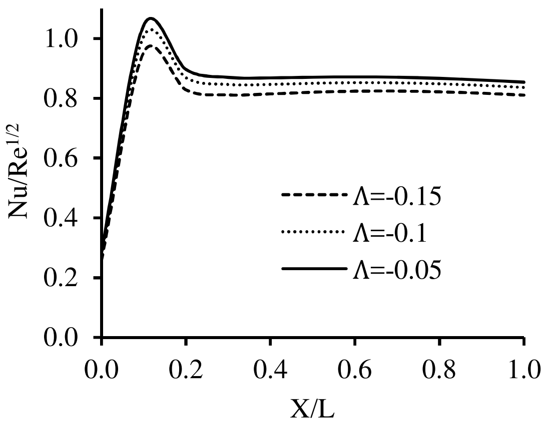

Figure 2: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = 0.6

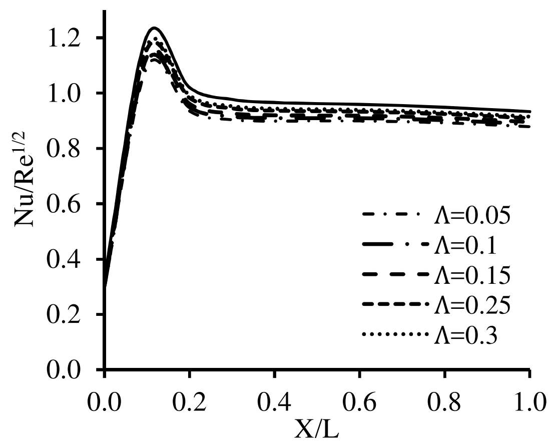

Figure 3: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = 0.6

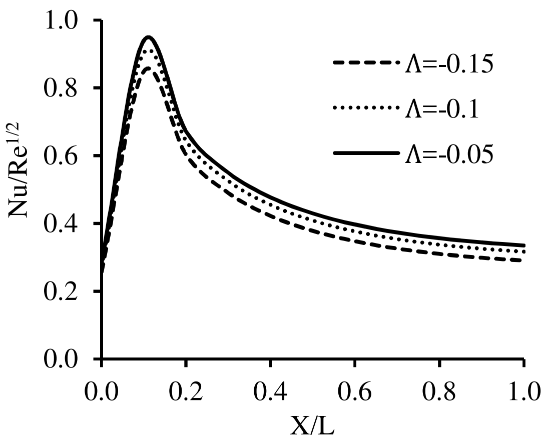

Figure 4: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.2

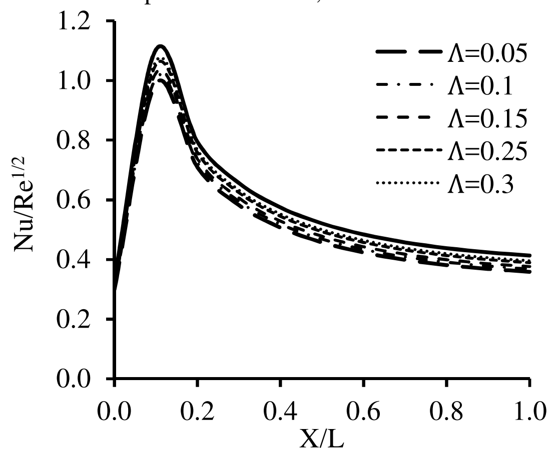

Figure 5: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.2

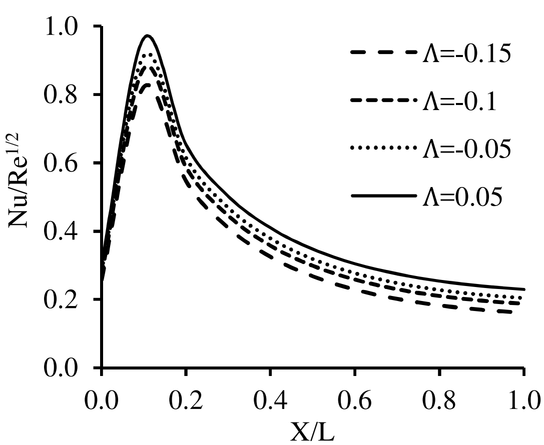

Figure 6: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.4

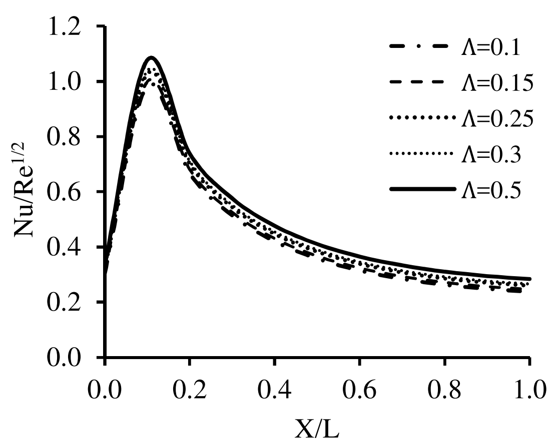

Figure 7: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.4

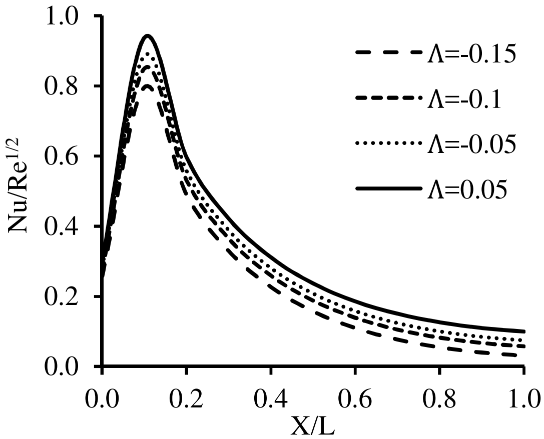

Figure 8: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.6

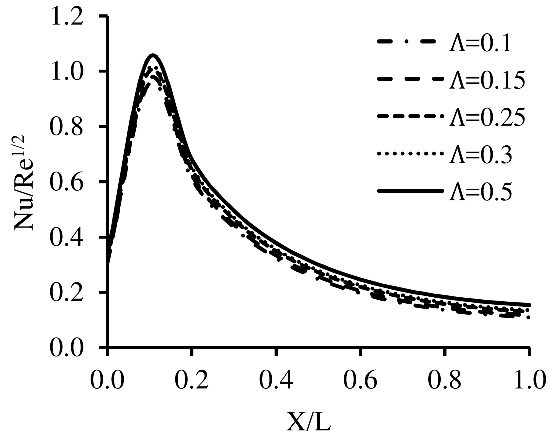

Figure 9: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.6

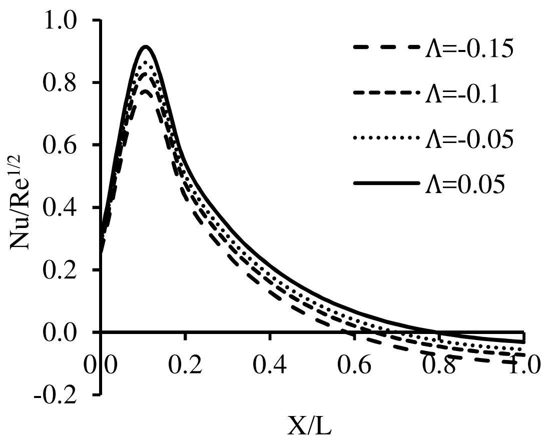

Figure 10: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.8

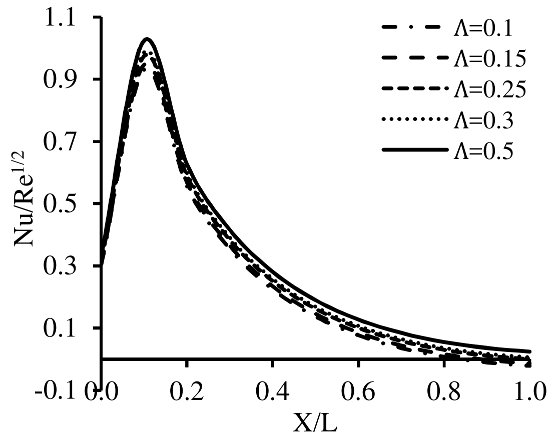

Figure 11: Heat transfer over a non-isothermal flat plate for Pr = 0.7, a = -0.8

4. CONCLUSION

The two-parameter Merk's series which was corrected by Chao and Fagbenle and applied to a constant wall temperature or isothermal surface has been applied to a variable or non-isothermal inclined flat plate. In addition to the MCF two-parameter problem, a third parameter known as the temperature parameter came up in the non-isothermal case. The temperature parameter was evaluated and incorporated into the MCF series and was used for the analysis of the flow and heat transfer over the inclined flat plate. Result were compared with that of Chao and Fagbenle at zero incidence, (Λ=0), and are found to be in good agreement.

5. ACKNOWLEDGMENT

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6. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

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