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**Review Article**

# Anatomization of Thick Rectangular Plates: A Critical Review

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**ABSTRACT**

*The use of plate materials such as structural steel is growing rapidly. Several studies have been conducted on the characteristics of thick plates using contrasting deflection functions, theories and methods to investigate their structural performance. The remarkable benefits of this structural materials to many engineering applications has grown its research horizon. A thorough review was done in this paper to keep a quick glance of this exceptional construction material. This paper describes elaborately bending, buckling and vibration behaviour of thick places. The knowledge of this reviewed works is crucial for framing engineering edifices. The outcome of this study will assist researchers and scientists to spot pertinent prevailing literatures quickly and enhance further studies. The elasticity theories employed by several scholars, the displacement functions, and the support conditions that were addressed, were encapsulated in this research work. Most studies reviewed in this paper focused on plates whose support-status were simply-supported at all the edges (SSSS) and clamped-supports at all the edges (CCCC). The outcome of this study based on review of previous studies showed that refined plate theories (RPTs) underestimates and over-anticipates stresses, displacements, frequencies and loads due to the supposition of deformation kinematics and negligence of the elements along the thickness-locus of the plate. This paper revealed that three-dimensional (3D)-plate theories are preferable and reliable since they produce accurate and exact solutions for thick plates. This work promotes sustainability in the construction industry and equips potential practitioners with adequate knowledge that strengthens choice-making on projects that require their utilization.*



## 1. INTRODUCTION

The need to ascertain accurate and reliable solutions to the behaviour of thick plates is intrinsic to sustainable infrastructural development due to its increasing demand and advancement of the construction industry. And bending, buckling and vibration analysis are the major areas of concentration among researchers on investigation of plates (Reddy, 2006). Assessment of these areas were covered in this paper.

Plates are 3-D structural materials with ordered proportions through y, x, and z pivots, whose depth is infinitesimal juxtaposed with the other plane and equidistant surfaces (Onyeka et al., 2023a). They are orthotropic, isotropic or anisotropic based on their material-configuration and deformation nature (Ventsel and Krauthammer, 2001; Onyeka *et al.,* 2023b). Their edge conditions can be simply-supported, fixed, or free. Hinged on shape, plates can be circular, triangular, elliptical, skew or rectangular. Plates are also categorized as thin, moderately-thick, or thick considering their thickness (Chandrashekhara, 2001; Onyeka *et al.,* 2022a). Predicated on the fraction of the depth (t) to the minimal lateral-dimension, rectangular plates with $45 \leq a/t \leq 100$ have been addressed as thin-plate, $20 \leq a/t \leq 45$ as comparatively-thick and $a/t \leq 10$ as thick plate where a/t is the span/depth correlation (Onyeka et al., 2022b). In this study, thick rectangular isotropic plates of different boundary conditions, are reviewed.

Thick plates have remained the interest of researchers for a very long period of time due to their cost benefits, lightweight, ability to be tailored to desired structural properties and load resistance characteristics (Onyeka *et al.*, 2021; Onyeka *et al.,* 2022c; Onyeka *et al.,* 2022d) with numerous applications in foundation footings, ship hulls, bulkheads, water tanks, spacecraft panels, bridge deck slabs, roof and floor slabs (Onyeka et al., 2022e)

Thick plates carry evenly-distributed loads and transverse loads on the plates' mid-plane. When it is exposed to an applied load at the edge sheer to the mid-surface and diffused through the plate's thickness, it is considered to be of uniform-distributed loading (Timoshenko and Woinowsky-krieger, 1970; Onyeka et al., 2022f) As a result of these loadings, plates become elastically deformed (Gujar and Ladhane 2015; Onyeka et al., 2022g). Without adequate management, plate deformation can result to structural failure. To maximize the properties of thick plates and to ensure economical and safe designs, several authors have investigated the attributes and failure conditions of these materials.

Different authors have employed varying theories to examine the attributes of thick-plates. Classical plate theory (CPT), the simplest plate model presented by Kirchhoff (Reddy, 1984; Whitney, 1987), is mostly applied in the analysis of thin plates as it neglects the transverse shear-deformation and assumes that the normal to the middle-plane before deformity stays straight and normal to the middle-surface after distortion. CPT cannot capture the exact behaviour of thick plates. 2D- theories (RPT) such as exponential shear deformation theory (ESDT), first order shear deformation plate theory (FSDT), trigonometric shear deformation theory (TSDT), Polynomial shear deformation theory (PSDT) and higher order shear deformation plate theories (HSDT) (Mantari and Soares 2012; Sayyad et al., 2013; Ghugal and Gajbhiye, 2016; Onyeka and Okeke, 2021) were introduced to overcome the limitations of CPT. These RPTs ignores the normal-strain/stress along the depth-axis of the plate, hence 3-D theory was introduced to capture this effect and to offer exact solutions for thick plates.

Thick plates are often analyzed using either numerical methods (approximate approach) or the analytical methods (closed-form approach) (Szilard, 2004; Onyeka et al., 2022c) Numerical method includes: finite element, finite-difference, Bubnov-Galerkin, variational (Galerkin, Ritz and Kantorovich), collocation and finite-strip techniques. Analytical methods include: Levy/Navier series, Eigen expansion methods, as well as the method of integral transforms. Several scholars have employed some of these methods (Osadebe et al., 2016; Nwoji et al., 2017; Onyeka et al., 2021b).

Researchers are continuously scrutinizing the characteristics of thick plates considering its growing demand and cost in the construction industry. Numerous studies have been carried out and more are ongoing. Thorough understanding of the displacements and stress-strain state of thick plates helps to ensure the reliability and strength of such structural materials. The aim of this review study is to epitomize antecedent research works on thick plate analysis. This paper focused on reviewing recent works. This study tends to reveal different models employed by several scholars to solve different plate issues. Hence this paper will produce the sufficient idea and premium information for other scholars working on plate materials and it is an obvious verdict that more studies is deserved to be done on sustainable structural development adopting thick plates as construction materials. This study is an answer to design problems on choice of plate supports in civil engineering works.

## 2. BENDING ANALYSIS OF THICK PLATES

Bending is the crosswise-distortion of plates towards their surface under the sway of moments and external forces. When these deformations exceed the critical applied load, plate failure occurs. In the investigation of plate structures, the major concern of scholars is to study the effect of this critical load since it largely determines the safety and stability of these structures.

Nwoji et al., (2021) developed a refined plate equation using Vasil'ev-approach and orthogonal-polynomial approach to resolve the bending-quandary of SSSS-plates exposed to uniformly distributed load. The authors observed the hold of transverse shear force on the deformation of elastic plates, which should not be disregarded. Due to inclusion of shear-deformation in the solution, the percentage difference between their study and Kirchhoff's hypotheses is high for fairly-thick plates (4.17% at h = 0.40 m).

Onyeka and Mama (2021) employed direct-variation technique, trigonometric shape functions with 3-D concept to ascertain the bending behaviour of thick-plates with simple-supports at all edges. At a span-depth ratio of 100, the outcome of their model almost corresponds with the values from the RPT and CPT. The authors considered thin plate to be one that whose span-depth ratio is 100 and above. The result obtained in their study differed with those of RPT and CPT by 21.1% and 25.4% respectively for the entire a/t ratios of 5, 10, 100 and 1000. But when juxtaposed with 3D-plate analysis, a percentage variation of 7.2% was recorded, which portrays the accuracy and reliability of 3D-plate model.

Exponential functions and RPT were used by Sayyad and Ghugal (2012) to investigate the bending hitch of SSSS-plates The authors obtained displacements including the stresses, destitute of correction-factor and their outcome was found adequate when collated with other RPTs. Their model under-estimates in-plane normal stress by 1.5%, PSDT under-estimates it by 3.7%, while TSDT amplified it by 4.1%, whereas HSDT and FSDT underestimates it by 3.7% and 9.2% respectively for aspect-ratio of 4. It was recorded that the values obtained from their model and other theories closely agreed with those of exact solution for aspect ratio of 10.

Ibearugbulem *et al.* (2021), addressed the bending issue of thick plates simply-supported at all the edges (SSSS), deriving exact polynomial-displacement-function from the governing-equation using an analytical approach with a 3-D plate concept. Their solutions gravitate to infinity. The amplitude of deflection obtained from their model differed from those of RPT by 26.63% and 5.01% for span-to-depth ratios of 4 and 1000 respectively, which showed that RPTs are quite unseemly for thick plate analysis, because it over-estimates the deflection and x-z plane shear stress.

Mama et al. (2017) applied finite Fourier-sine-transform technique to solve SSSS Kirchhoff plate bending problem under point load, general distributed load and uniform load. Closed-form solutions were obtained by the authors using the analytical method. Their solutions were equivalent to the solutions obtained with Navier's double trigonometrical series-method.

Although the strain and stress along the thickness-direction of the plate was not addressed, Mantari and Soares (2012) obtained a Navier-type analytical solution by employing HSDT with virtual-work concept and premise of variation in the mechanical properties of the plates in the thickness-locus, for SSSS-plates carrying transverse bi-sinusoidal loads.

Zhong and Xu (2017) developed the governing expression for the bending investigation of thick plates with all round clamped supports (CCCC) adopting Mindlin's plate HSDT-model. The complexity associated with deriving the coefficients of the plates were reduced using improved Navier's solution with partial differential equations and decoupling approach. Their studies showed identical outcome when compared with that of previous scholars. Their model has varied pragmatic-applications such as foundation-design of high-rise structures and rigid-pavements of highway and airports. The approach employed by authors also can be adopted to analytically solve plate boundary-issues such as point and spring supports.

Polynomial displacement function of fourth order HSDT and direct-variational technique was used by Ibearugbulem and Onyeka (2020) to decipher bending of clamped rectangular plates. Their result vouchsafes that the non-dimensional figures of in-plane-displacement parameters, (u and v), and that of out-of-plane displacement characteristics, (w) decrease as the span-depth thickness increases. For dimensional displacement features, the variation of the out-of-plane quantities reduced as the span-thickness ratio increased and became insignificant from span-thickness ratio of 15. The values of non-dimensional form of deflection for span-thickness ratios of 20 and above were equal to the value from CPT, which uncloaked the frontier between thin and thick plate. At 87 % confidence level, the results obtained from their work were the equivalent to those from of previous studies.

3-D ideology was used by Grigorenko *et al.* (2013) to study the bending behaviour of thick plates with clamped edges conditions using the numerical approach. Two-coordinate directions of spline-collocation and discrete-orthogonalization method of solving higher-order differential equations were considered and the resultant stresses and displacements in the plates were satisfactory. Their study showed that the deflection function is strongly determined by the plate geometry and the support conditions. Deflection decreases where the length of one side increases and the deflection of plates with clamped edges are less than that of the same plate with hinged supports.

Ibearugbulem *et al.* (2018) analyzed the bending solution of CCCC-plates adopting the polynomial deflection function and shear deformation theory. The model developed by the authors were based on Ritz-energy method and the principle of elasticity was used to prepare the total potential energy analogy of the plate. When the total potential energy relationship was subjected to direct variation, the governing expressions for evaluating displacement coefficients were obtained. The authors idealized values of non-dimensional vertical-shear-stress ($\boldsymbol{\tau_{xz}}$) between span-depth ratios (α) of 60 and 100, as thin plate, those between 20 and 50 as moderately thick plates and those less than 15, as thick plates.

Onyeka *et al.,* (2022c) considered the bending of plates clamped at two adjacent edges and the others simply supported (CSCS), applying 3-D theory with trigonometric-function with the aim of obtaining a close-form (exact) solution. The authors employed an analytical approach to design the methodology. Their study showed that growth in the aspect-ratio of the plate decreases the in-plane displacements along x and y axis (u and v) whereas, the deflection (w) which occurs in the plate because the applied-load rises with enlargement in the magnitude of the a/t ratio of the plate. The stress perpendicular to the x, y and z axis proliferates as the span-depth ratio of the plate increases. The a/t ratio addressed were 4, 5, 10, 15, 20, 50, 100 and CPT.

Onyeka *et al.,* (2022h, 2022i, 2023a, 2023b) applied energy-method to address the bending problems of CCCS, CSSS, SCFS and SSFS thick plates respectively. The authors analyzed the shear distortion upshot in the spatial facet along x, y and z axes. Exact shape-functions were used. In (Onyeka et al., 2023a-2023b), polynomial functions were adopted while in (Onyeka *et al.*, 2022h; Onyeka *et al.*, 2022i], trigonometric functions were used. The percentage-variation between Onyeka et al (2022i) and those of 2-D (RPT) with an assumed-displacement and 2-D -RPT with derived function is 1.43% and 5.15% respectively. Onyeka et al.(2022h) differed with CPT and RPT of assumed deflection by 5.6% and 4.9% respectively, whereas it varied from the exact 2-D RPT by 3.7%.

Ghugal and Gajbhiye (2016) obtained a closed-form analytical solutions for simply-supported thick plates made prone to distributed single-sinusoidal loads. The authors applied virtual work principle to obtain the governing equations and boundary conditions of 5$^{th}$ -order HSDT. The authors considered the effect of transverse-shear and normal-strain deformation. The displacement functions used were assumed and accounted for the non-linear variation of in-plane displacements. The theory presented by the authors under-predicts the transverse-displacement slightly for aspect-ratio 4 compared to other RPTs and the exact solution, while their results of in-plane displacements and normal stress are very close to those of RPTs and exact theory. The investigations on bending of thick plates were presented in Table 1.

Table 1: Critical review summary on bending of thick plates

| Authors | Theories employed | Method of analysis | Deflection function used | Plates considered |
|---|---|---|---|---|
| Nwoji *et al.,* (2021) | RPT | Vasil'ev approach | Polynomial | SSSS |
| Onyeka *et al.,* (2023a) | 3-D plate theory | Energy method | Polynomial | SCFS |
| Sayyad and Ghugal (2012) | RPT | Principle of virtual work | Exponential | SSSS |
| Grigorenko *et al.* (2013) | 3-D Plate theory | Spline-collocation method & variable-separation technique | Trigonometric | CCCC |
| Ghugal and Gajbhiye (2016) | RPT | Virtual work principle | Trigonometric | SSSS |
| Ibearugbulem *et al.* (2018) | RPT | Ritz energy method | Polynomial | CCCC |
| Onyeka *et al.,* (2022c) | 3-D Plate theory | Variational principle | Trigonometric | CSCS |
| Ibearugbulem *et al.* (2021) | 3-D Plate theory | Analytical approach | Polynomial | SSSS |
| Onyeka and Mama (2021) | 3-D Plate theory | Direct variational technique | Trigonometric | SSSS |
| Mama *et al* (2017) | CPT | Finite Fourier sine transform method | Trigonometric | SSSS |

## 3. BUCKLING ANALYSIS OF THICK PLATES

Structural instability is commonly noticed through buckling and this is a case where a structure encounters significant distortion and can no longer sustain its ability to resist the load at a critical-load value. Many researchers have analyzed the stability of thick plates considering its increasing demand in the design of engineering structures.

The stability of simply-supported thick isotropic plates under biaxial and uniaxial in-plane forces, were analyzed by Sayyad and Ghugal (2012) using refined plate theory and an assumed exponential function. The idea of virtual-work was employed to obtain the governing equations and related support-conditions of the model. The value of critical buckling load increased with increase in aspect-ratio. Aspect ratios (a/h) of 5, 10, 20, 50 and 100 were addressed in their study. According to their findings, the buckling load is critical when mode for the plate is (1, 1), where the plate is subjected to uniaxial and biaxial compression, while buckling load is critical when mode for the plate is (1, 2) for plates subjected to tension in x-direction and compression in y-direction.

The stability of a uniaxially compressed CSCS rectangular thick plates was investigated by Onyeka *et al.,* (2022j) using 3-D plate concept. The authors obtained the general and direct governing-equation of the plate using the general and direct variation of the total potential energy function. The plates' deflection was the product of the coefficient of deflection and shape-function of the plate. The exact deflection of the plate was obtained using the derived trigonometric and polynomial shape functions. Through the variation of the whole potential energy expression, the authors obtained the model for the critical buckling load and other expression. Exact solutions were derived from trigonometric functions than those of polynomial with a variation of 2.4%.

Onyeka *et al.,* (2022b) employed 3-D approach to address the stability of plates clamped at the first and second edges and the other edges, free and simply supported respectively (CCFS). From the equations of compatibility, the authors obtained the exact shape functions and the deflection coefficient of the plate derived from the governing equation. The trigonometric model predicted a slightly higher value of the critical buckling load than polynomial function at an aspect ratio of 1.0, 1.5, 2.0, 2.5 whereas the polynomial-model predicts a slightly higher value of the critical buckling load than trigonometric function at an aspect ratio of 3.0, 3.5, 4.0, 4.5 5.0. Higher value of stiffness coefficient was obtained with trigonometric function than polynomial. The authors concluded that trigonometric displacement-function is safe and more accurate for analysis of thick plates as they offer close-form solutions.

Ezeh *et al.,* (2019) probed into the buckling of isotropic SSSS-plates, harnessing polynomial shear-deformation and displacement functions. The maximum and minimum percentage difference between their study and that of Gunjal et al., (2015) were 2.20% and 0.04% at span-depth ratio of 5 and 50 respectively, while the upmost and least percentage variation between their model and that of Sayyad and Ghugal (2012) were 2.33% and 0.00% at span-thickness ratio of 5 and 100 respectively, which shows that enlarging the span-depth ratio makes their results closer to Sayyad and Ghugal (2012). These variations being less than 5% validates their study as a good model for analyzing buckling of thick plates.

Gunjal *et al.,* (2015) used virtual-work concept and trigonometric-functions to study the buckling of thick SSSS-plates subjected to biaxial and uniaxial compression. The authors considered ratios of b/a of 1.0, 1.5, 2.0, 2.5, 3.0, 3.5 and 4.0 with various aspect ratios a/h of 5, 10, 20, 50 and 100, where length "a" is in x-direction, width "b" in y-direction and thickness "h" in z-direction. The authors observed that when the plate was subjected to uniaxial compression along x-axis, increase in b/a ratios decreases the critical buckling load while it was elevated with increase in a/h ratios. When the plate was subjected to uniaxial-compression along y-axis, critical buckling load is increased with increase in b/a ratios and a/h ratios.

Variables-separation technique was employed by Moslemi *et al.* (2016) with the augmented Love's displacement potential function to solve the buckling matter of thick plates simply supported at all the edges (SSSS). The plates were subjected to biaxial and uniaxial static loads. Displacement-potential functions were used to simplify the governing equations into two partial differential equations ($2^{nd}$ order and $4^{th}$ order). Their study showed that the buckling load and critical buckling mode increases with increasing of plates' thickness. Their study showed that as the thickness of the plate gets stretched, the buckling factor mode diminishes and the compression load in y-axis reduces the critical buckling load and mode.

A polynomial shape function and 3-D theory of elasticity were used by Onyeka *et al.,* (2022a) to ascertain the stability of thick plates with free-support at its third edge and simply-supported at other edges (SSFS). The plates were subjected to a uniaxial compressive load. The functional for total potential energy was obtained using the 3-D constitutive relations. The authors developed exceptional formula for ascertaining the critical-buckling-load and their study showed that failure in a plate structure is bound to happen as the in-plane load on the plate rises and gets critical.

Differential Quadrature (DQ) method was applied by Zhang and Wang (2011) to evaluate buckling. The transverse shear effect was taken into consideration using the Mindlin-plate-theory. The authors also employed both the incremental and deformation model to obtain the buckling stress factors. under either uniaxial or equi-biaxial compressions. The model generally generated consistently lower buckling loads than the incremental-theory and there is large discrepancy in predictions between two theories exists with increasing of the plate thickness, E/σ0, and c in the Ramberg–Osgood relations.

3-D elasticity theory and variational-method with the blend of trigonometric and polynomial function, were employed by Onyeka *et al.,* (2022e), to analyze the stability of thick plates clamped on the first edge, free at the third edge, with the second and fourth edges simply-supported respectively (CSFS) and the effect of aspect-ratio on the critical buckling load of the plate. The 3-D model established by the authors gave desired results for both functions and yielded exact solutions compared to solutions of RPT. The authors presented the percentage variation of critical buckling load between their model using polynomial, and that of trigonometric function for an isotropic. The gross average percentage differences between the two functions were recorded to be 6.4% which is quite acceptable in statistical analysis as it is below 10%. Trigonometric-function offered close-form solution than the polynomial.

Saheb and Aruna (2015) considered coupled-displacement-field (CDF) method to evaluate the buckling load parameters of the moderately thick rectangular plates. The authors employed a single-term trigonometric function with assumption that it is for the total rotations (in both X; Y directions). The method they adopted is one whose coupling equation is independent of the applied load. The authors considered SSSS-thick plates subjected to biaxial and uniaxial compressive loads at different aspect ratios. Generally, it was noted that increase in the plate thickness-ratio led to decrease in the non-dimensional buckling loads. Table 2 captured a brief summary on buckling of thick plates.

Table 2: Critical review summary on buckling of thick plates

| Authors | Theories employed | Method of analysis | Deflection function used | Plates considered |
|---|---|---|---|---|
| Onyeka *et al*., (2022b) | 3-D plate theory | Direct variational calculus | Polynomial and trigonometric | CCFS |
| Moslemi *et al.,* (2016) | 3-D theory | Separation of variables method | Displacement potential | SSSS |
| Onyeka *et al*., (2022a) | 3-D elasticity theory | Energy method | Polynomial | SSFS |
| Ezeh *et al.,* (2018) | RPT | Numerical approach | Polynomial | SSSS |
| Sayyad and Ghugal (2012) | RPT | Virtual-work approach | Exponential | SSSS |
| Onyeka *et al*., (2022e) | 3-D plate theory | Variational method | Trigonometric and polynomial | CSFS |
| Gunjal *et al.,* (2015) | RPT | Principle of virtual work | Trigonometric | SSSS |

## 4. VIBRATION ANALYSIS OF THICK PLATES

Onyechere *et al.,* (2020a) employed polynomial-deflection function with a HSDT to analyze the vibration of thick plates whose supports were of CSCS, CSSS and CCCS conditions. The outcome of their study proved that the ability of the plate to resist vibratory-load reduces as the ratio of the in-plane dimensions (b/a) increases. Their results were in consonance with previous studies.

A third-order shear deformation concept was adopted by Hosseini-Hashemi et al. (2011) to examine the free-vibration of plates with opposite edges simply-supported, blending hyperbolic and trigonometric displacement functions. The equations of motion and natural support-conditions of the plate was derived from Hamilton's concept. Due to the effect of rotary inertia and shear deformations, the thickness-length ratio was more noticed for natural frequencies with higher-mode. Their study revealed that higher-constraints at the supports expand the flexural rigidity of the plate which leads to higher frequency response.

Lin and Shi (2019) considered a 3-D concept with a blend of trigonometric and polynomial functions while Hashemi and Arsanjani (2005) used amalgamation of trigonometric and hyperbolic functions. Lin and Shi (2019) studied the vibration of plates placed on elastic-foundations with varied support-conditions and Hashemi and Arsanjani (2005) derived exact characteristic equations for vibrating moderately thick rectangular plates of classical boundary state.

Gajbhiye *et al.* (2021) applied $5^{th}$ order shear-deformation theory for free vibration of SSSS-plates. The concept of virtual work was employed to obtain the fundamental equations and support-conditions. Navier-solution approach was adopted to ascertain the thickness stretch frequencies and MATLAB program was used to solve the eigenvalue problem. The fundamental mode bending frequencies predicted by the theory were analogous to those of exact analysis. It was observed that TSDT and HSDT over-estimates and under-estimates the bending frequency for basic and higher-modes respectively, while FSDT produced the least values of bending frequency for all vibration modes.

Onyechere *et al.,* (2020b) used polynomial deflection functions and a PSDT to study the free-vibration of SSSS-plates. The authors derived a simple linear equation suitable for generating the fundamental natural-frequency parameters at varied aspect-ratio of the plate. As the ratio (a/t) decreases, the authors recorded a reduction in the value of the non-dimensional natural frequency parameter at the same value of 'P'. At any value of 'a/t', there was a reduction in the value of the non-dimensional natural frequency parameter as 'P' rises, having the highest value at P = 1 (square plate). Where "P" was the aspect ratios (b/a) of 1.0, 1.2, 1.4, 1.6, 1.8, 2.0, 2.2 and 2.4. Span-thickness ratios considered were 5, 10, 15, 20, 25,30, 40, 50, 60,70, 80, 90 and 100.

Sayyad *et al.,* (2015) presented a TSDT for vibration-assessment of isotropic thick-plates. In their numerical study, they considered SSSS-plates and Navier's solution approach was applied for the analytical solution. The authors obtained the fundamental-frequencies using the Finite Element Method in ANSYS. The authors among other theories chose results obtained with RPT and compared those solutions with results from ANSYS. The model produced results with little error.

Huang et al (2011) used Mindlin-plate-model and Ritz-method to study the nodal patterns and the frequencies of thick simply-supported plates. The authors developed sets of admissible-functions for Ritz system. New sets of admissible-functions were proposed to constitute the behaviours of true solutions along the crack, as well as augmenting the potentiality of Ritz approach in handling cracked plates. The increase in the quantity of admissible functions forms exact frequencies as upper-bounds. Side and internal cracks were the only cracked configuration addressed by the authors.

The dimensionless frequencies of thick plates whose boundary order were SCSC, SSSS, and SCSS, were obtained by Zargaripoor *et al.,* (2018), using HSDT and wave-propagation mechanism. SCSC and SSSS had the topmost and minimum frequency values respectively. The authors derived the matrices of reflection and propagation first and their combination produced the governing equation of the plate. In their study, it was observed that plates with SCSC and SSSS boundary-condition had the apical and lowest frequency values respectively. And the rate of frequency reduction for SCSC-plates was larger. The frequency-ratio reduces with increasing aspect-ratio for constant value of (h/a) ratio. The dimensionless frequency values for varied values of (b/a) ratio decreased as the thickness ratio increased. It was observed that the rate of frequency reduction was low for higher values of (b/a) ratio.

Ibearugbulem *et al.* (2020) applied polynomial-shape function and energy method, with 2-D -RPT to analyze the stability of thick plates. The authors ascertained the critical load, fundamental natural and linear frequency by replacing the polynomial-function into the governing-expression and the outcome was compared with those of FSDT. The authors analogized the fundamental linear frequencies of vibration of orthotropic SSSS and SSFS plates. For a/t ratio of 10, the fundamental-linear-frequency for orthotropic SSFS-plate corresponding to modulus of elasticity ratios ($E_1/E_2$) of 10, 25 and 40 are 0.0016, 0.0022 and 0.0026 Hz. The corresponding values using FSDT are 0.0015, 0.0021 and 0.0025 Hz. The percentage variation with the exact value is 1.07%. A maximum percentage difference of 1.29% was recorded, when compared with other RPTs. The absence of shear-deformation parameters contributed to the variation in the results. In Table 3, a brief summary of studies on vibration of thick plates was presented.

Table 3: Critical review summary on vibration of thick plates

| Authors | Theories employed | Method of analysis | Deflection function used | Plates considered |
|---|---|---|---|---|
| Gajbhiye *et al.*(2021) | RPT | Virtual work concept and Navier-solution technique | Exponential | SSSS |
| Onyechere *et al.,*(2020a) | RPT | Numerical approach | Polynomial | CSCS, CSSS and CCCS |
| Sayyad *et al.*(2015) | RPT | Navier's solution approach | Trigonometric | SSSS |
| Onyechere *et al.* (2020b) | PSDT | Energy method | Polynomial | SSSS |
| Ibearugbulem *et al.* (2020) | RPT | Ritz method | Polynomial | SSSS and SSFS |

Lately, most scholars adopted non-classical elasticity theories to investigate the behaviour of plates (Onyeka et al., 2022c; Mousavi Khoram et al., 2019). Surface theory, couple-stress theory, nonlocal-elasticity theory, strain-gradient theory and nonlocal strain-gradient model have been applied to analyze nanostructures (Noroozi et al., 2020; Mohammadi et al., 2022). Applying consistent couple-stress theory and nonlocal elasticity theory, Nejad *et al* (2016; 2017; 2018) have performed studies on elasticity, buckling and vibrations of beams made of functionally graded materials (FGM). To analyze FGMs, nonlocal-strain-gradient elasticity theory and Hamilton's principle were employed by Hadi et al., (2018), Shishesaz and Hosseni (2019), and Shishesaz et al (2017) while Couple-stress theory was employed by Gorgani et al., (2019) and Barati et al., (2020) to capture the size effects of FGMs. The elasticity of FGM structures of variable thickness was also presented by Hosseini *et al* (2016; 2019). Mohammadi et al (2013a; 2013b; 2013c), also presented studies on the elasticity, vibration and buckling behavior of nano structured plates and beams.

## 5. CONCLUSION

Thick plates are one of the common and feasible construction materials whose research have been explored in different fronts in Nigeria with encouraging results. Based on the review of some published studies presented, the study concluded that many studies were based on the application of refined plate theories which made assumptions about the kinematics of deformation and neglects the stress along the thickness axis of the plate. RPTs underestimates and over-predicts stresses, displacements as well as the bending loads. Few scholars employed and confirmed that 3-D plate theories are preferable and reliable because they offer accurate and exact solutions for thick plates. More investigations by different authors will boost this discovery. Previous scholars who analyzed thick plates limited their study on SSSS, CCCC, CSCS plates. There is need for more investigation on other plates with supports like CSSS, CCCS, SSFS, SCFS, SCFC, CCFC, CSFS and CCFS plates. Although numerical method of analysis is not tedious, it generates approximate results and the stresses and displacements at any point in the plate cannot be evaluated. The analytical technique adequately satisfies the governing expression in the plate domain and across the supports of the plate. Unlike assumed shape functions, exact displacement shape functions derived from the governing equilibrium equation yield closed-form solutions and ensures cost-effective and safe analysis of thick plates. And trigonometric functions give exact solutions incongruous with exponential, polynomial, and hyperbolic whose exact function gravitate to infinity. Understanding the methods, theories and functions to be adopted in investigation of plate materials such as structural steel, will enhance sustainability in the construction industry.

## 6. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

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