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A Model for Multi-Distribution Policy for Perishable Products in a Supply Chain Network

Orsarh, E.S., Oluleye, A.E., *Anyaeche, C.O., Adeosun, P.O. and Kalio, N.T.

Department of Industrial and Production Engineering, University of Ibadan, Ibadan, Nigeria.
*osita.anyaeche@ui.edu.ng

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ABSTRACT

Modern markets across the globe have witnessed a high expedition of consumers and volatility of demand. As such, business owners are refocusing their investments toward their customers and suppliers for production efficiency and marketability. Industries handling deteriorating products such as agro-food are faced with the sparseness of models that suit the nature of their industry, especially with price variance mostly in developing countries. This study was conducted to develop a multi-supply chain distribution model for perishable products in a price-invariant setting which aims to minimize wastage from the leftovers. In the example problem, the firm produces products with shelf life of four days and sold to distributors who adopt ordering policies of one-day, two-day, three-day and four-day and buy 40, 30, 20 and 30 % of available quantity of products, respectively. The model provided a production plan that aligns with the demand for products, which in turn is driven by the time-related ordering policies of the distributors. They order to minimize losses given the product's short shelf life and zero salvage values at the end of shelf life. The model and the example given demonstrate a production plan that aligns well with demand.

1. INTRODUCTION

Modern markets across the globe have witnessed a high expedition of consumers and volatility of demand (YoKell, 2014). As such, business owners are refocusing their investments toward their customers and suppliers for production efficiency and marketability. Management techniques now adopt more collaborations and partnerships that are responsive to the needs of consumers. To maintain healthy synergies across these networks, manufacturers are beginning to appreciate the supply chain as a dependable marketing strategy. Indeed, the supply chain—the system of production and distribution of products across networks/the matrix of suppliers and demand markets—is favorable for manufacturers to manage the synergies effectively. However, without proper management of the supply chain itself, there could be supply chain inefficiencies that lead to profit loss due to volatile demand forecasts, known as the bullwhip effect

(Khosroshahi, et al, 2016; Braz et al, 2018). Therefore, Supply chain management, which is the management of a business's supply chain to curtail bullwhip effects and thereby maximize manufacturer's profits, becomes a vital management tool.

As with other industries the Agro-food industry, the businesses that handle deteriorating and perishable products are increasingly realizing that they have no point operating in an island with the increasing volatility of demand as mentioned earlier. Thus, they are adapting global organizations with the new trend—with distribution across developing and developed countries (Kandil, *et al.*, 2020). Problems arising from these modern trends attract even more investigations through modeling, analysis, and computations in the area of the supply chain (Kandil et al, 2020), to increase their certainty of efficiency across supply chain integrations.

The agro-food firms are faced with the sparseness of models that suit the nature of their industry (Lucas and Chhajed, 2004; Ahumada and Villalobos, 2009; Yu and Nagurney, 2013). Extant observations indicate that this is critical for the profitability of the already thin profits of the industry (Yu and Nagurney, 2013). This is unlike the profitability of other industries which have better-adapted supply chain management models.

Along the supply chain, perishable products experience continuous and significant deterioration in quality, with no further use but to be discarded after a period. This strains the supply chain for perishable products to involve large product wastes and loss of profits (Yu and Nagurney, 2013). Amongst others, Nagurney and Yu (2013), Rijpkema et al, (2014), Balaji and Arshinder (2016) have explored supply chain networks either with a specific focus on one or two drivers (Rijpkema et al, 2014) or with a broad approach to the whole supply chain of perishable products (Yu and Nagurney, 2013). However, more work in this area can further shed clearer light on the efficiency of existing models, via analytic improvements and the development of novel concepts. Some work in this direction includes, Samiha, *et al.*, (2022), Lihong, et al, (2023) and Al-Ashhab and Fahad Alanazi, (2024).

Samiha *et al.*, (2022) research proposes a tri-objective optimisation model for multi-echelon and multi-products aiming to lessen the annual supply chain cost, and cold storage setup cost, and enhance the freshness of perishable by establishing a proper distribution channel. The multi-criteria problem, a weighted sum method is considered and solved using CPLEX ie Concert Powerful EXtreme Linear Programming Solver, optimisation studio. It is designed to solve a wide range of optimization problems, including linear programming, mixed integer programming, quadratic programming, and quadratically constrained programming. CPLEX provides flexible and robust algorithms for solving complex optimization problems efficiently and accurately. The feasibility of the model with two common fruits of Bangladesh, i.e. guava and lemon. Finally, several cost-effective options and trade-offs between three factors are presented to aid the decision-making process.

Lihong, et al, (2023), focuses on optimizing the long- and short-term planning of the perishable product supply chain network (PPSCN). It addresses the integration of strategic location, tactical inventory, and operational routing decisions. The main objective is to minimize the overall supply chain cost using nonlinear mixed integer programming model is developed for the multi-echelon, multi-product, and multi-period location-inventory-routing problem (LIRP) in the PPSCN. Two hybrid metaheuristic algorithms, namely genetic algorithm (GA) and multiple population genetic algorithm (MPGA), are hybridized with variable neighborhood search (VNS) and proposed to solve this NP-hard problem. Sensitivity analysis is conducted to examine the influence of key model parameters on the optimal objective, providing valuable management implications. The results validate the efficacy of the proposed model and solution method as a reliable tool for optimizing the design problem of the PPSCN.

Al-Ashhab and Fahad Alanazi, (2024), developed a mathematical model for perishable products that aims to maximize total profit in addition to preventing the expiration of perishable products using the FIFO inventory strategy to reduce environmental impact by reducing waste. It is worthy to note in all these, price invariance was not the focus.

The existing models for perishable products accommodating price variance or strategic pricing nature of Nagurney's models still meet certain other limitations. In developing countries, prices do not vary with the deteriorative state of produce over time. As a traditional practice, produce prices stay the same over their shelf-life until they are consumed or have expired (Aworh, 2020). Indeed, product differentiation informs pricing decisions in more structured climes with more sophisticated and regulated supply chain networks. This fosters non-cooperative oligopolistic competition, transfer prices, and consumer safety practices that encourage variation in prices along the supply chain network due to deterioration in the developed economies (Kogan and Herbon, 2008; Li and Nagurney 2015; Liu and Papageorgiou, 2018).

The picture is different with developing economies such as Africa which operates without an organized market structure but with inflexible transfer prices and high competition amongst middlemen. The commonness of complacent preservation and consumer services practices, prices have no room to vary except by undeveloped market forces or contingencies (Aworh, 2020). Whereas these proactive and developed practices are underway in emerging economies, they are yet to be significantly integrated into developing economies (Esfahbodi and Zhang, 2019). This results in more sensitivity with distributors and retailers to the days a product has spent on the shelf. In other words, the expected behavior is sensitive to decision-makers' response to time. Even the most advanced models for perishable goods and price variance are at odds in capturing the new behavior at price invariance. Thus, neither profitability nor the efficiency of supply chain management could be captured efficiently under the existing models in the context of developing economies.

Supply chain management aims to satisfy customers by striking a balance between efficient delivery and responsiveness that fits the organizational strategy to accomplish the goals. In a typical supply chain, the drivers include inventory, transportation, facilities, information, sourcing, and pricing (Shahzadi, *et al.*, 2013). The drivers combine to make it possible to achieve the results of getting the product to the demand markets and the final revenue.

There have been studies such as Masoumi, *et al.* (2012); Chen, *et al.* (2015); and even; Samiha *et al.*, (2022); Lihong, et al, 2023 and Al-Ashhab and Fahad Alanazi, 2024 that have covered the supply chain networks of perishable products where prices are responsive to deterioration over time. However, in developing economies, the demand for price invariant models for deteriorating, perishable produce is what can capture the scenario, suggesting that this bridge in the literature is yet to be covered. A possible explanation is that it is most logical that supply chain management approaches deterioration with price variation. This holds in developed economies but not in developing economies (Aworh, 2020).

The primary objective of this study is to develop a model for multi-policy supply chain distribution network for perishable products in a price-invariant setting. The focus of the work is on modeling deteriorative products where the supply chain network is driven by time-related ordering policies of distributors.

2. METHODOLOGY

2.1. Model Assumptions

This work is based on the following assumptions.

i. The prices are invariant of the products and remain constant throughout their shelf lives
ii. Expired products no longer have economic value
iii. Products are first shipped to the distributors and then to the retailers according to their demands.
iv. The distributors and retailers exhaust their products before more orders are placed.
v. In the process of distribution, the distributors are given priority according to their policies.

2.2. Transactions by Distributors' Policies

The transaction between the manufacturer and distributors is guided by a production plan. This plan includes the distribution strategy by which the manufacturer decides to produce according to the distributors' product policies to minimize wastage from the leftovers. There is the maximum acceptable number of days the distributors accept products as well as the percentage of goods to take. Each in this order adopts a policy; hence we model the following policy periods for the manufacturer and distributor transactions.

Priority: For any given day of production, the one-day policy buyer is severed before the two-day buyer, and the two-day buyer is severed before the three-day buyer, just as the three-day buyer is severed before the four-day buyer.

2.2.1. One-day life policy

Distributors with one-day life policy place and accept t_{j1} proportion of the products that are freshly produced. They accept products that have stayed less than a day, with the manufacturer, after production. These products are shipped immediately to these distributors without delays. The quantity of product *j* sold to distributors with one-day policy is given as:

$$Q_{j11} = t_{j1} \times Q_{j1} \tag{1}$$

$$Q_{j11left} = (1 - t_{j1}) \times Q_{j1} \tag{2}$$

Where k_1 is distributor with one day purchasing policy; t_{j1} is the portion of product Q_{j1} purchased by k_1; Q_{j1} is the quantity of product j produced on day 1; Q_{j11} is the quantity of product *j* sold to distributors with one-day policy; $Q_{j11left}$ is the quantity of the product left after k_1 has purchased on day 1.

2.2.2. Two-day life policy

Distributors with a two-day life policy place and accept t_{j2} proportion of products that are freshly produced and leftovers from the previous day. They accept products that have stayed less than 2 days with the manufacturer.

Let Q_{j2} = the quantity of the product produced on day-2

From Equation (2) the quantity of the product left from day-1 is given as:

$$Q_{j11left} = (1 - t_{j1}) \times Q_{j1}$$

The quantity of product *j* produced on day-2, sold to distributors with one-day policy is given in Equation (3) as:

$$Q_{j21} = t_{j1} \times Q_{j2} \tag{3}$$

$Q_{21,left}$, the quantity of the product left after k_1 buys is given as :

$$Q_{21left} = (1 - t_{j1}) \times Q_{j2} \tag{4}$$

The proportion of products that distributor with 2-day policy, k_2, buys is t_{j2}

The quantity of the product sold to the distributors with 2-day policy periods, Q_{j22}, has two components: ie from the leftover from day 1 and day 2, after one day buyer has bought, and is given as:

$$Q_{j22} = t_{j2}\left[(1 - t_{j1})Q_{j1} + (1 - t_{j1})Q_{j2}\right] = t_{j2}(1 - t_{j1})\left[Q_{j1} + Q_{j2}\right] \tag{5}$$

The quantity left of the produc Q_{j22} left after k2 has purchased is givenas:

$$Q_{j22\,left} = (1 - t_{j2})\left[(1 - t_{j1})Q_{j1} + (1 - t_{j1})Q_{j2}\right] = (1 - t_{j2})(1 - t_{j1})\left[Q_{j1} + Q_{j2}\right] \tag{6}$$

2.2.3. Three-day life policy

The distributors with a three-day life policy place and accept t_{j3} proportion of the products produced two previous days and the freshly produced products of the day's production, after k_2 has been served. These products have stayed less than three days with the manufacturer. Production quantity on day 3 is given as Q_{j3}

The quantity left on day 2 from Equation (6):

$$Q_{j22\,left} = (1 - t_{j2})(1 - t_{j1})\left[Q_{j1} + Q_{j2}\right]$$

Thus, Q_{j31}, the quantity of product *j* produced on day-3 sold to distributor k_1 on the one-day policy is given as:

$$Q_{j31} = t_{j1} \times Q_{j3} \quad (7)$$

$Q_{j31,\,left}$ the quantity of the product left after k_1 has purchased on day-3 is given as

$$Q_{j31,\,left} = (1 - t_{j1}) \times Q_{j3} \quad (8)$$

Thus, the quantity of product *j* produced on day 3 and sold to distributors with a two-day policy Q_{j32} is given as:

$$Q_{j32} = t_{j2}(1 - t_{j1})[Q_{j3} + (1 - t_{j2})Q_{j2}] \quad (9)$$

The quantity that is left after the purchase by k_2 i.e. $Q_{j32,\,left}$ is given as:

$$Q_{j32,\,left} = (1 - t_{j2})(1 - t_{j1})[Q_{j3} + (1 - t_{j2})Q_{j2}] \quad (10)$$

Similarly, the product purchased by distributors with the 3-day policy period is derived as:

$$Qj_{33} = t_{j3}\{(1 - t_{j2})(1 - t_{j1})[Q_{j3} + (1 - t_{j2})Q_{j2} + Q_{j1}]\} \quad (11)$$

By similar logic, the quantity left in day 3 after k_3 has purchased, $Q_{j33,left}$ is expressed as:

$$Q_{j33,\,left} = (1 - t_{j3})\{(1 - t_{j2})(1 - t_{j1})[Q_{j3} + (1 - t_{j2})Q_{j2} + Q_{j1}]\} \quad (12)$$

A careful observation of Equations (1) to (12) of what is bought, and what is left suggests that these equations are recursive in nature.

2.3. Recursive Equations

Similar computations are done for subsequent days of production. Distributor with k-day life policy places and accepts products that were produced *k-1* days together with the freshly produced products of the day's production. They accept products that have stayed less than k-days with the manufacturer. Thus, the quantity of the product produced on the day k is expressed as Q_{jk}.

Thus what is left of the day *k-1* day is given as $Q_{jk-1,left}$ is given in (12) as:

$$Q_{jk-1,left} = (1 - t_{jk-1})\{(1 - t_{jk-2}) \ldots (1 - t_{j1})[Q_{jk-1} + \cdots . + (1 - t_{jk-2}) \ldots \ldots (1 - t_{j2})Q_{j2} + Q_{j1}]\} \quad (13)$$

The quantity of product *j* produced on k-day and sold to distributors with a one-day policy is

$$Q_{jk1} = t_{j1} \times Q_{jk} \quad (14)$$

Similarly, the quantity of product left after k1 has bought, $Q_{jk1,left}$ is given as:

$$Q_{jk1,left} = (1 - t_{j1}) \times Q_{jk} \quad (15)$$

Likewise, the quantity of product *j* produced on day k and sold to distributors with a two-day use policy is this

$$Q_{jk2} = t_{j2}\{(1 - t_{jk-1})(1 - t_{jk-2}) \ldots \ldots (1 - t_{j1})[Q_{jk-1}]\} \quad (16)$$

The quantity of the product left is

$$Q_{jk2,left} = (1 - t_{j2})\{(1 - t_{jk-1})(1 - t_{jk-2}) \ldots \ldots (1 - t_{j1})[Q_{jk-1}]\} \quad (17)$$

The quantity of product *j* produced on day k, sold to distributors with (k-1)-day policy

$$Q_{jk-1} = t_{jk-1}\{(1-t_{jk-1})(1-t_{jk-2})\ldots\ldots(1-t_{j1})[Q_{jk-1} + (1-t_{j2})Q_{jk-2} + \cdots\ldots + (1-t_{jk-2})\ldots\ldots(1-t_{j2})Q_{j2}]\} \quad (18)$$

Quantity of the product left on day k

$$Q_{jk-1,left} = (1-t_{jk-1})\{(1-t_{jk-1})(1-t_{jk-2})\ldots\ldots(1-t_{j1})[Q_{jk-1} + (1-t_{j2})Q_{jk-2} + \cdots\ldots\ldots + (1-t_{jk-2})\ldots\ldots(1-t_{j2})Q_{j2}]\} \quad (19)$$

2.4. The Company

A firm manufactures a daily quantity Q_{ji} of a perishable product that has a shelf life of four days. If there are four distributors within its supply chain network, that buy according to the given policies, As part of the initial conditions, not more than 50% capacity of its 8200 units per day is utilized in its first 4-day-cycle of production, based on the shelf life of the product. For the first cycle of production, we consider the production for the first four days as Q_{j1} = 1000, Q_{j2} = 2000, Q_{j3} = 4000 and Q_{j4} = 3200 units for days 1, 2 3, and 4 respectively. For these initial conditions, we need to establish the daily quantities the firm should produce to meet the policy demands of the distributors and minimize wastes resulting from expired products. The firm produces Q_{ji} quantity of product j daily. It sells the products to four distributors, k_1, k_2, k_3 and k_4. The distributors buy their proportions, t_{jk} of the available products. These proportions are: k_1 (t_{j1} = 0.4), k_2 (t_{j2} = 0.3), k_3 (t_{j3} = 0.2) and k_4 (t_{j4} = 0.3). The sequence of service is that k_1 is served before k_2, k_2 is served before k_3, k_3 is served before k_4. The k_i buys only t_{ji} percentage of the current day's production quantity. Given a 4-day cycle of production due to a 4-day shelf life, it is required to determine the quantity of products purchased by each of the distributors according to its policy and precedence requirement; and the total products left after the shelf life of the products has expired. This, however, may change due to logistics. The scenarios that may results due to policy changes are not considered as well as other variants of the policies as these are not the focus of this work. However, they are going concerns. For example, the proportions may vary, and may also be unknown. Also, purchases preferences may vary.

2.5. Model Application

To facilitate the utility of the model presented in section 2.2-2.4 Equations (1) to (19) an Excel Template is generated and presented as in Tables 1 and 2. The model application examines the case where the percentage demands of the distributors are known to the manufacturer and are constant. Other scenarios, e.g. varying, unknown or different variants are not considered but and are going concerns elsewhere.

2.5.1. Data for example problem

The data for the four-day production for the first four days i.e.: Q_{j1} = 1000, Q_{j2} = 2000, Q_{j3} = 4000 and Q_{j4} = 3200 units for day 1, 2 3 and 4 respectively and the purchases by k_1 (t_{j1} = 0.4), k_2 (t_{j2} = 0.3), k_3 (t_{j3} = 0.2) and k_4 (t_{j4} = 0.3) are applied in Tables 1 and 2.

2.5.2. The excel template

Table 1, shows the Excel template of the model showing the formulae formulation of the problem using the model developed in Equations (1) to (19). As shown in Table 1, there are nine columns. Column 1 is the serial number, followed by the days of production and the buyers. C, D are for the proportion bought according to the policies. E is for quantity produced per day, while G, H, I, and J are the quantities bought by the distributors in days 1, 2, 3, and 4 respectively. For clarity, the table has been split into two; Tables 1 and 2, representing the analysis for the four days production took place within the production cycle and Table 2 represent when production cycle has ended but products are yet to expire. This helps to closely show the distributors make purchases after production cycle (four days) and to monitor the product shelf life. Note that the item produced in day 1 expired after day 4, while those produced on day 4 expired in day 7. Thus the segmentations to show purchases till day 7 when all the products within the given production cycle expired.

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Table 1: Excel template for days of production

3	B	C	D	E	G	H	I	J
4				Quantity				
5	Buyers	%age		Produced	Q1	Q2	Q3	Q4
6	Day 1			Q_j =				
7	Purchase by k1	t_{j1} =	0.4	1000	=D7*E7			
8	Qty left by k1				=E7-F7			
9	**Day 2**			Q_j =				
10	Purchase by k1	t_{j1} =	0.4	2000	=D10*E10	=E10-F10		
11	Qty left by k1				=E7-F7			
12	Purchase by k2	t_{j2} =	0.3		=D12*F11	=D12*G10		
13	Qty left by k2				=F11-F12	=G10-G12		
14	Day 3			4000				
15	Purchase by k1	t_{j1} =	0.4	4000	=D15*E15			
16	Qty left by k1				=F13	=G13	=E15-F15	
17	Purchase by k2	t_{j2} =	0.3		0	=D17*G16	=D17*H16	
18	Qty left by k2				=F16	=G16-G17	=H16-H17	
19	Purchase by k3	t_{j3} =	0.2		=D19*F18	=D19*G18	=D19*H18	
20	Qty left by k3				=F18-F19	=G18-G19	=H18-H19	
21	**Day 4**							
22	Purchase by k1	t_{j1} =	0.4	3200	=D22*E22			
23	Qty left by k1				=F20	=G20	=H20	=E22-F22
24	Purchase by k2	t_{j2} =	0.3		0	0	=D24*H23	=D24*I23
25	Qty left by k2				=F23	=G23	=H23-H24	=I23-I24
26	Purchase by k3	t_{j3} =	0.2		0	=D26*G25	=D26*H25	=D26*I25
27	Qty left by k3				=F23	=G25-G26	=H25-H26	=I25-I26
28	Purchase by k4	t_{j4} =	0.3		=D28*F27	=D28*G27	=D28*H27	=D28*I27
29	Qty left by k4				=F27-F28	=G27-G28	=H27-H28	=I27-I28

After day four, the first set of expired products emerged. These are shown in cell G29. In Day 5, k_1 will not buy because there are no fresh products. The k_2 will however buy from only day 4 products, while k3 will buy from both day-3 and day-4 products. The k_4 distributor will buy from day-2, day-3, and day-4 products. Similar logic is used for the rest of the table while the quantities of the expired products are as shown in G64, H64 , I64 and J64 for Q_{j1}, Q_{j2}, Q_{j3} and Q_{j4} respectively.

Table 2: Excel template from end of production days till product expiry dates

3	B	C	D	E	G	H	I	J
4								
5	Buyers	%age		Qty	Q1	Q2	Q3	Q4
38	Day 5							=F27-F28
39	Purchase by k1	t_{j1} =	0.4		0	= G27-G28	= H27-H28	=I27-I28
40	Qty left by k1				=F29	0	0	0
41	Purchase by k2	t_{j2} =	0.3		0	=G29	=H29	=I29
42	Qty left by k2				=F29	0	0	=D41*I29
43	Purchase by k3	t_{j3} =	0.2		0	=G29	=H29	=I29-I41
44	Qty left by k3				=F42	0	=D43*H42	=D43*I42
45	Purchase by k4	t_{j4} =	0.3		0	=G42	=H42-H43	=I42-I43
46	Qty left by k4				=F44	=D45*G44	=D45*H44	=D45*I44

47	Day 6			k3 and k4 will buy on day6	=G44-G45	=H44-H45	=I44-I45
48	Purchase by k1	t_{j1} =	0.4				0
49	Qty left by k1			=F46	0	0	0
50	Purchase by k2	t_{j2} =	0.3	0	=G46	=H46	=I46
51	Qty left by k2			=F49	0	0	=D50*I46
52	Purchase by k3	t_{j3} =	0.2	0	=G49	=H49	=I49
53	Qty left by k3			=F51	0	0	=D52*I51
54	Purchase by k4	t_{j4} =	0.3	0	=G51	=H51-H52	=I51-I52
55	Qty left by k4			=F53	0	=D54*H53	=D54*I53
56	Day 7			k4 will buy on day7	=G53-G54	=H53-H54	=I53-I54
57	Purchase by k1	t_{j1} =	0.4				0
58	Qty left by k1			=F55	0	0	0
59	Purchase by k2	t_{j2} =	0.3	0	=G55	=H55	=I55
60	Qty left by k2			=F58	0	0	0
61	Purchase by k3	t_{j3} =	0.2	0	=G58	=H58	=I58
62	Qty left by k3			=F60	0	0	0
63	Purchase by k4	t_{j4} =	0.3	0	=G60	=H60-H61	=I60-I61
64	Qty left by k4			=F62	0	0	=D63*I62
	After day 7 all				=G62-G63	=H62-H63	=I62-I63
		Wastes					=F64/E7
	%age wastes			=100*F66	=G64/E10	=H64/E15	=I64/E22

3. RESULTS AND DISCUSSION

The results of the application of the template is shown in Tables 3 and 8. Tables 4 through 7 are the respective purchases by the distributors, while 9 through 12 are the show the purchases after the production cycle has ended. In Table 4, the quantities on the different days of production are 1000, 2000, 4000 and 3200 for the day 1, 2, 3 and 4 respectively. The quantities purchased by k_1 are 400, 800, 1600 and 1280 on days 1, 2, 3 and 4 respectively. Observe that the first production cycle ended on day-4 according to this analysis. These represent 40% of production on those days. In Table 5 the purchses by k_2 are shown. Observe that the components are made according to its policy, as 180 of day 1 production and 360 of day two production. In day 4, k_2 purchased 403 of day 3 production and 576 of day 4 production. The total quantity of day 2 production purchased by k_2 is 612. Similar explanations hold for others. Table 6 shows the purchases made by k_3 distributor. These are 84, 118, 336 of Q_{j1}, Q_{j2} and Q_{j3} respectively, representing the three relevant days that met its policy. Observe that these represent the three relevant ages of the products. As can be seen from the Table 7, k_4 bought a total of 101, 113, 226 and 323 of Q_{j1}, Q_{j2}, Q_{j3} and Q_{j4} respectively., because of its policy. These purchases are made after k_1, k_2 and k_3 have been served in order to maintain the precedence relationships. Also, k_4 is the only distributor that has products spanning the four days sheilf life in keeping with its policy.

Day-5 to day-7 Purchases

Next we consider purchases made after the production cycle has ended, ie day-5, day-6 and day-7, given a 4-day cycle of production due to 4-day shelf life. These are as shown in Table 8. Tables 9 to 11. Observe that k_1 will not make purchases on day-5 because no new products were made. However, k_2, k_3 and k_4 will buy on day-5, k_3 and k_4 will buy on day-6, k_3 and k_4 will buy on day 6, while only k_4 will buy on day 7. Furthermoe, the quantities of expired products are shown in Table 12. Observe that k_1 distributor did not purchase after the end of the production cycle, as pointed out earlier. In Table 9, we show the purchase of 226 made by k_2 on day-5. Observe that this purchse was made from only Q_{j4} which was onday old as the policy allows. Others are not allowed. From Table 10, observe that k_3 bought 105 of Q_{j3} and 105 of Q_{j4}, representing what it s policy allows, because they still met its policy requirements. As far as k_3 is concerned, it cannot buy from Q_{j1} which were still available. On day-6 it still its policy allows the purchase of 59 units of product from Q_{j4}. As can be seen from the Table 11, k_4 bought 79 of Q_{j2} , 126 of Q_{j3} and 126 of Q_{j4} ton day-5.Note that k2 could still buy from all except Q_{j1} which has expired on day-5. Again note that k4 made purchases on both day-6 where it bought 89 from Q_{j3} and 71 from Q_{j4}. A poosble advantage here is that the compny may encourage k_4 do do clearance purchase. It is only k_4 that made purchase on the day 7, ie 50 from Q_{j4} day-4 production, just before the product expired the following day, because of its policy.

Table 3: Excel Template for days of Production

3	B	C	D	E	G	H	I	J
4	Production			Quantity				
5	Buyers	%age		Produced	Q1	Q2	Q3	Q4
6	Day 1			Q_j = 1000				
7	Purchase by k1	t_{j1} =	0.4	1000	400			
8	Qty left by k1				600			
9	**Day 2**			Q_j = 2000				
10	Purchase by k1	t_{j1} =	0.4	2000	800	1200		
11	Qty left by k1				600			
12	Purchase by k2	t_{j2} =	0.3		180	360		
13	Qty left by k2				420	840		
14	Day 3			4000				
15	Purchase by k1	t_{j1} =	0.4	4000	1600			
16	Qty left by k1				420	840	2400	
17	Purchase by k2	t_{j2} =	0.3		0	252	720	
18	Qty left by k2				420	588	1680	
19	Purchase by k3	t_{j3} =	0.2		84	118	336	
20	Qty left by k3				336	470	1344	
21	**Day 4**							
22	Purchase by k1	t_{j1} =	0.4	3200	1280			
23	Qty left by k1				336	470	1344	1920
24	Purchase by k2	t_{j2} =	0.3		0	0	403	576
25	Qty left by k2				336	470	941	1344
26	Purchase by k3	t_{j3} =	0.2		0	94	188	269
27	Qty left by k3				336	376	753	1075
28	Purchase by k4	t_{j4} =	0.3		101	113	226	323
29	Qty left by k4				235	263	527	753

Table 4: Purchases by distributor k_1

			Day 1	Day 2	Day 3	Day 4
k_1	Q_{j1}	1000	400			
	Q_{j2}	2000		800		
	Q_{j3}	4000			1600	
	Q_{j4}	3200				1280

Table 5: Purchases by distributor k_2

		Production	Day 1	Day 2	Day 3	Day 4
k_2	Q_{j1}	1000		180		
	Q_{j2}	2000		360	252	
	Q_{j3}	4000			720	403
	Q_{j4}	3200				576

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Table 6: Purchases by distributor k_3

		Production	Day 1	Day 2	Day 3	Day 4
k3	Q_{j1}	1000			84	
	Q_{j2}	2000			118	94
	Q_{j3}	4000			336	188
	Q_{j4}	3200				269

Table 7: Purchases by distributor k_4

		Production	Day 1	Day 2	Day 3	Day 4
k_4	Q_{j1}	1000				101
	Q_{j2}	2000				113
	Q_{j3}	4000				226
	Q_{j4}	3200				323

Table 8: Excel Template from end of production days till product expiry dates

	Buyers	%age		Q1	Q2	Q3	Q4
29	Qty left by k4			235	263	527	753
38	Day 5			235	263	527	753
39	Purchase by k1	t_{j1} =	0.4	0	0	0	0
40	Qty left by k1			235	263	527	753
41	Purchase by k2	t_{j2} =	0.3	0	0	0	226
42	Qty left by k2			235	263	527	527
43	Purchase by k3	t_{j3} =	0.2	0	0	105	105
44	Qty left by k3			235	263	421	421
45	Purchase by k4	t_{j4} =	0.3	0	79	126	126
46	Qty left by k4			235	184	295	295
47	Day 6			k3 and k4 will buy on day 6,			
48	Purchase by k1	t_{j1} =	0.4	0	0	0	0
49	Qty left by k1			235	184	295	295
50	Purchase by k2	t_{j2} =	0.3	0	0	0	89
51	Qty left by k2			235	184	295	295
52	Purchase by k3	t_{j3} =	0.2	0	0	0	59
53	Qty left by k3			235	184	295	236
54	Purchase by k4	t_{j4} =	0.3	0	0	89	71
55	Qty left by k4			235	184	207	165
56	Day 7			k4 will buy on day 7			
57	Purchase by k1	t_{j1} =	0.4	0	0	0	0
58	Qty left by k1			235	184	207	165
59	Purchase by k2	t_{j2} =	0.3	0	0	0	0
60	Qty left by k2			235	184	207	165
61	Purchase by k3	t_{j3} =	0.2	0	0	0	0
62	Qty left by k3			235	184	207	165
	Purchase by k4	t_{j4} =	0.3	0	0	0	50
	Qty left by k4			235	184	207	116
	After day 7 all products are expired						
	Wastes			0.24	0.09	0.05	0.04
	%age wastes			24	9	5	4

Table 9: Purchases by distributor k_2 after the end of the production cycle

k_2	Q_{j1}	1000	Day 5	Day 6	Day 7
	Q_{j2}	2000			
	Q_{j3}	4000			
	Q_{j4}	3200			
			226		

Table 10: Purchases by distributor k_3 after the end of the production cycle

k_3	Q_{j1}	1000	Day 5	Day 6	Day 7
	Q_{j2}	2000			
	Q_{j3}	4000			
	Q_{j4}	3200	105		
			105	59	

Table 11: Purchases by distributor k_4 after the end of the production cycle

k_4	Q_{j1}	1000	Day 5	Day 6	Day 7
	Q_{j2}	2000			
	Q_{j3}	4000	79		
	Q_{j4}	3200	126	89	
			126	71	50

Expired Products

As can be seen from Table 12, 235, 184, 207 and 116 of Q_{j1}, Q_{j2}, Q_{j3} and Q_{j4}.andrespectively were not urchase having exceeded their expiry dates. These represent 24, 9 5 and 4% of the products on the respective days. This information is a very veriable guide for management in planning its production in order to minimize waste.

Table 12: Quantities left at the expiry of all the products.

Q_{ty}	Q_{j1}	Q_{j2}	Q_{j3}	Q_{j4}
Qty left by k_4 after day-7 (products expiry day)	235	184	207	116
Wastes	0.24	0.09	0.05	0.04
%age wastes	24	9	5	4

Tables 4, 5, 6 and 7 show the purchases made by the distributors by k_1 (t_{j1} = 0.4), k_2 (t_{j2} = 0.3), k_3 (t_{j3} = 0.2) and k_4 (t_{j4} = 0.3) on the different days of production.The transaction model between the manufacturer and distributors is developed to guide production plan and the distributors' products policies which aims to minimize wastages from the leftovers. For a four-day production, the firm produced Q_{j1} = 1000, Q_{j2} = 2000, Q_{j3} = 4000 and Q_{j4} = 3200 and the distributors. The quantities left after Day 4: Q_{j1} =235, Q_{j2} = 263, Q_{j3} =527, Q_{j4} = 753. These result to 24, 13.15, 13.17 and 23.53%. The total quantities bought by k_1, , k_2 , k_3 and k_4 are as shown in the referenced tables. In comparism, aAt the end of day seven, the leftover was Qty left after Day 7:235, 184, 207 and 116 of products Qj1, Qj2, Qj3 and Qj4 respectively. This translates to Q_{j1} =24, Q_{j2} = 9, Q_{j3} =6, Q_{j4} =4%. This multi-distribution policy for distributing perishable products to the various distributors provides a framework for optimizing the production process to minimize waste from expired products based on the distributors policy. This is similar to Gharehyakheh *et al.* (2020) multi-objective model for optimizing the distribution process for perishable products, considering the products' freshness, vehicle emissions and distribution costs to reduce product wastage. The shelf life has significant impact on the quality of perishable products which in turn influence the production plan of the organisation to ensure that the products get to the final consumer as early as possible within the expiration period. Considering the inventory levels after each day of production, the quantity of products to hold is a function of the distributor's policy. Proportions of daily production volume and leftovers are purchased implying daily decreasing inventory levels. focus on consolidation of production, inventory and distribution processes. Dolgui *et al.* (2018) proposed similar approach using a multi-stage supply chain integrated inventory policy for perishable considering production, inventory and distribution processes to minimize waste and ensure timely delivery of fresh products to the distributors. Aazami & Saidi-Mehrabad (2021) described this approach as a multi-period production-distribution planning model suitable for perishable products with fixed shelf lives. This model aids production optimization, distribution decisions and inventory management to maximize both the manufacturer's revenue and the seller's profit. Existing models among which are, Dolgui *et al.* (2018), Gharehyakheh *et al.* (2020), and Aazami and Saidi-Mehrabad (2021); addressed shipping policies of distributing products from the various plants in a period to the distributors by ensuring shipping the maximum possible quantity of units to minimise distribution cost and wastage without considering the distributors' demand policies. The proposed model relaxes the assumption of existing demands from the distributors by considering the distributors' precedence, demand, and purchasing policies allowing for flexibility in their choices in the supply chain. In particular, the work emphasizes these factors:

i. Precedence Relationship: We need to ensure that distributors follow a precedence relationship, i.e., distributor k_i can only buy if distributor k_{i-1} has bought their share.
ii. Age of Product Policy: Distributors might have different policies regarding the age of the product they are willing to purchase. This policy determines the maximum acceptable age h_k for each distributor k.
iii. Quantity Purchase Policy: Each distributor k might have a specific proportion t_k of the total production Q_j they are willing to purchase.

4. CONCLUSION

The developed model, along with the illustrated example, showcases an effective production strategy tailored to meet demands with varying policies. It highlights the potential for minimizing losses among a group of distributors with diverse purchasing policies. A production plan spanning four days has been showcased, tailored to accommodate the perishable nature of the firm's products with a four-day shelf life. The development of a transaction model between the manufacturer and distributors serves as a blueprint for both the production plan and the distributors' product policies, with the overarching goal of reducing wastage from unsold inventory. Through an illustrative example, various purchases made by distributors across different production days have been detailed. The model can be used to plan production given a set of distributors and their purchasing policies and the production capacity of a manufacturing plant. It can also be used for right sizing the manufacturing plant or for planning a new plant. These are planninig and thus will allow the manager to adjust the production accordingly.

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6. CONFLICT OF INTEREST

There is no conflict of interest associated with this work.

REFERENCES

Aazami, A.& Saidi-Mehrabad, M. (2021). A production and distribution planning of perishable products with a fixed lifetime under vertical competition in the seller-buyer systems: A real-world application. *Journal of Manufacturing Systems*, 58 (Part A), pp. 223-247

Ahumada, J. O., & Villalobos, R. (2009). Application of planning models in the agri-food supply chain: A review. *European Journal of Operational Research*, 196(1), pp. 1-20. doi: 10.1016/j.ejor.2008.02.014

Al-Ashhab, M. S. and Fahad A. (2024). Developing a multi-item, multi-product, and multi-period supply chain network design and planning model for perishable products. *International Journal of Research*, 10(4), pp. 179-199

Aworh, C. O. (2021). Food safety issues in fresh produce supply chain with particular reference to sub-Saharan Africa. *Food Control*, 123(7). doi:10.1016/j.foodcont.2020.107737

Balaji, M., & Ashinder, K. (2016). Modeling the causes of food wastage in Indian perishable food supply chain. Resources, Conservation and Recyling, 114, pp. 153-167. doi:https://doi.org/10.1016/j.resconrec.2016.07.016

Braz, A. C., Mello, A. M., Vasconcelos Gomes, L.A., & Nascimento, P. T. (2018). The bullwhip effect in closed-loop supply chains: A systematic literature review. *Journal of Cleaner Production*, 202, pp. 376-389. doi:https://doi.org/10.1016/j.jclepro.2018.08.042

Chen, S. C., Teng, J. T., & Skouri, K. (2014). Economic production quantity models for deteriorating items with up-stream full trade credit and down-stream partial trade credit. *International Journal of Production Economics*, 155 (1), pp. 302-309. doi:https://doi.org/10.1016/j.ijpe.2013.07.024

Dolgui, A., Tiwari, M. K., Sinjana, Y., Kumar, S. K., Son, Y. (2018). Optimising integrated inventory policy for perishable items in a multi-stage supply chain. *International Journal of Production Research*, 56 (1-2), pp. 902 - 925.

Esfahbodi, A., & Zhang, Y. (2020). Sustainable Supply Chain Management in Developed vs. Emerging Economies: Evidence From the UK and China's Manufacturing Industry. Encyclopedia of Renewable and Sustainable Materials, 5(1), pp. 537-549. doi:10.1016/B978-0-12-803581-8.11506-1

Gharehyakheh, A., Krejci, C.C., Cantu, J., and Rogers, K.J. (2020). A Multi-Objective Model for Sustainable Perishable Food Distribution Considering the Impact of Temperature on Vehicle Emissions and Product Shelf Life. *Sustainability*, 12(16), 6668. https://doi.org/10.3390/su12166668

Kandil, N., Battaia, O., & Hammami, R. (2020). Globalisation vs. Slowbalisation: a literature review of analytical models for sourcing decisions in supply chain management. *Annual Reviews in Control*, 49, pp. 277-287. doi:https://doi.org/10.1016/j.arcontrol.2020.04.004

Khosroshahi, H., Husseini, S. M., & Marjani, M. (2016). The bullwhip effect in a 3-stage supply chain considering multiple retailers using a moving average method for demand forecasting. *Applied Mathematical Modelling*, 40(21), pp. 8934-8951. doi:https://doi.org/10.1016/j.apm.2016.05.033

Kogan, K., & Herbon, A. (2008). A supply chain under limited-time promotion: The effect of customer sensitivity. *European Journal of Operational Reasearch*, 188(1), pp. 273-292. doi:https://doi.org/10.1016/j.ejor.2007.04.012

Li, D., & Nagurney, A. (2015). Supply chain performance assessment and supplier and component importance identification in a general competitive multitiered supply chain network model. *Journal of Global Optimization,* 67(1-2), pp. 223-250.

Lihong, P., Miyuan, S. & Linfeng, L. (2023). Optimizing Perishable Product Supply Chain Network Using Hybrid Metaheuristic Algorithms. Sustainability, 15(3), pp. 1-21. https://doi.org/10.3390/su151310711

Liu, S., & Papageorgiou, L. G. (2018). Fair profit distribution in multi-echelon supply chains via transfer prices. *Omega, 80*, pp. 77-94. doi:https://doi.org/10.1016/j.omega.2017.08.010

Lucas, M. T. & Chhajed, D. (2004). Application of Location Analysis in Agriculture: A Survey. Journal of the Operations Research Society, Palmgrave Macmillan, 55(6), pp. 561-578

Masoumi, A. H., Yu, M., & Nagurney, A. (2012). A supply chain generalized network oligopoly model for pharmaceuticals under brand differentiation and perishability. *Transportation Research Part E: Logistics and Transportation Review,* 48(4), pp. 762-780. doi:https://doi.org/10.1016/j.tre.2012.01.001

Rijpkema, W. A., Rossi, R., & Vorst., J. V. (2014). Effective sourcing strategies for perishable product supply chains. *International Journal of Physical Distribution and Logistics* , 44(6), pp. 494-510.

Samiha M. J., Sudipta, D, Autoshe, R. C., S. A. & Ripon, K. C. (2022). Multi-objective multi-echelon distribution planning for perishable goods supply chain: a case study. *International Journal of Systems Science: Operations & Logistics,* 10(1).

Shahzadi, I., Amin, S., & Chaudhary, K. M. (2013). Drivers of Supply Chain Performance Enhancing Organizational Output: An Exploratory Study for Manufacturing Sector. *European Journal of Business Management*, 5(4), pp. 53-64.

YoKell, B. (2014). The Four Arguments - In Demand Planning and Forecasting. *Supply Chain Management* Review [Online] Retreived from: https://www.scmr.com/article/the_Four_Arguments_In_Demand_Planning_and_Forecasting [Accessed: 6th June, 2024]

Yu, M., & Nagurney, A. (2013). Competitive food supply chain networks with application to fresh produce. *European Journal of Operational Research*, 224(2), pp. 273-282. doi:https://doi.org/10.1016/j.ejor.2012.07.033.